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# A refined higher-order shear deformation theory for bending, vibration and buckling analysis of functionally graded sandwich plates

Kien T. Nguyen<sup>\*1</sup>, Tai H. Thai<sup>2</sup> and Thuc P. Vo<sup>3</sup>

<sup>1</sup>*Faculty of Civil Engineering and Applied Mechanics, University of Technical Education Ho Chi Minh City,*

*1 Vo Van Ngan Street, Thu Duc District, Ho Chi Minh City, Viet Nam*

<sup>2</sup>*School of Civil and Environmental Engineering, The University of New South Wales,*

*NSW 2052, Australia*

<sup>3</sup>*Faculty of Engineering and Environment, Northumbria University,*

*Newcastle upon Tyne, NE1 8ST, UK*

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**Abstract.** A refined higher-order shear deformation theory for bending, vibration and buckling analysis of functionally graded sandwich plates is presented in this paper. It contains only four unknowns, accounts for a hyperbolic distribution of transverse shear stress and satisfies the traction free boundary conditions. Equations of motion are derived from Hamilton's principle. The Navier-type and finite element solutions are derived for plate with simply-supported and various boundary conditions, respectively. Numerical examples are presented for functionally graded sandwich plates with homogeneous hardcore and softcore to verify the validity of the developed theory. It is observed that the present theory with four unknowns predicts the response accurately and efficiently.

**Keywords:** functionally graded sandwich plates; bending; buckling; vibration

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## 1. Introduction

Functionally graded (FG) sandwich structures are advanced composite materials that have smooth variations of material properties to eliminate the stress concentrations at the interfaces

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<sup>\*</sup>Corresponding author, PhD Kien T. Nguyen, E-mail: ntkien@hcmute.edu.vn

between faces and core found in conventional sandwich structures. These advanced structures are recently developed for various engineering applications where strong stiff and light structures are required. Typically, the FG sandwich plate faces are made from a mixture of ceramic and metal constituents which are continuously varied while the core is fully homogeneous material.

Increase of FG material applications in engineering structures has led to the development of many plate theories to predict accurately the bending, buckling and vibration behaviours of FG plates. The classical plate theory (CPT) (Feldman and Aboudi 1997, Javaheri and Eslami 2002, Mahdavian 2009, Mohammadi *et al.* 2010, Chen *et al.* 2006, Baferani *et al.* 2011) yields acceptable results only for the thin plates, whereas accuracy of the first-order shear deformation theory (FSDT) (Mohammadi *et al.* 2010, Croce and Venini 2004, Efraim and Eisenberger 2007, Zhao *et al.* 2009a, b, Lee *et al.* 2009, Hosseini-Hashemi *et al.* 2011, Naderi and Saidi 2010, Nguyen-Xuan *et al.* 2012, Thai and Choi 2013a) depends on the shear correction factor. Higher-order shear deformation theories with five unknown functions, which are included third-order shear deformation plate theory (TSDT), sinusoidal shear deformation plate theory (SSDT), hyperbolic shear deformable plate theory (HSDT), exponential shear deformation plate theory (ESDT), predict more accurate the response of moderate and thick FG plates (Reddy 2000, Zenkour 2006, Matsunaga 2008, Chen *et al.* 2009, Pradyumna and Bandyopadhyay 2008, Gilhooley *et al.* 2007, Talha and Singh 2010, Mantari and Soares 2012, 2013, Neves *et al.* 2012a, b, Jha *et al.* 2013, Thai and Kim 2013, Thai and Choi 2013b, Zenkour 2013). Some studies on response of FG sandwich plates have been carried out using higher-order shear deformation theories. Hamidi *et al.* (2012) and Abdelaziz *et al.* (2011) studied bending response, while Meiche *et al.* (2011) investigated vibration and buckling analysis of FG sandwich plates. Sobhy (2013) examined the vibration and buckling behavior of exponential FG sandwich plates resting on elastic foundations under various boundary conditions. By using quasi-3D higher-order shear deformation theories, which the stretching and shear deformation effects are taken into account, Neves *et al.* (2012c) and Zenkour (2013) investigated bending analysis, while Bessaim *et al.* (2013) focused on both bending and free vibration of FG sandwich plates. A n-order shear deformation theory and a 3D linear theory of elasticity were proposed by Xiang *et al.* (2011) and Li *et al.* (2008) for free vibration of FG sandwich plates. Literature surveys reveal that although there are some research works reported on FG sandwich plates, the studies on bending, buckling and vibration responses of these structures in a unified fashion are a few in number. By using different shear deformation theories, Zenkour (2005a, b) investigated bending, vibration and buckling problem of sandwich plates with FG faces and homogeneous hardcore. These problems were also solved by Neves *et al.* (2013) by using a quasi-3D high-order shear deformation theory and a meshless technique. Thai *et al.* (2014) presented a new FSDT for sandwich plates composed of FG face sheets and an isotropic homogeneous core.

The objective of this study is to propose a higher-order shear deformation theory for bending, vibration and buckling analysis of FG sandwich plates in a unified fashion. The proposed theory contains only four unknowns, accounts for a hyperbolic distribution of transverse shear stress and satisfies the traction free boundary conditions. Equations of motion are derived from Hamilton's principle. The Navier-type and finite element solutions are derived for plate with simply-supported and various boundary conditions, respectively. Numerical results are obtained for FG sandwich plates with homogeneous hardcore and softcore to investigate the effects of the power-law index, thickness ratio of layers and side-to-thickness ratio on the deflections, stresses, critical buckling loads and natural frequencies.

## 2. Problem formulation

Consider a three-layer sandwich plate as in Fig. 1. The face layers are made of a ceramic-metal material and the core layer is constituted by an isotropic material. The vertical positions of the bottom and top surfaces, and of two interfaces between the layers are denoted by  $h_0 = -h/2, h_1, h_2, h_3 = h/2$ , respectively. Here,  $h$  is the plate thickness,  $h_1, h_2$  vary according the thickness ratio of layers.

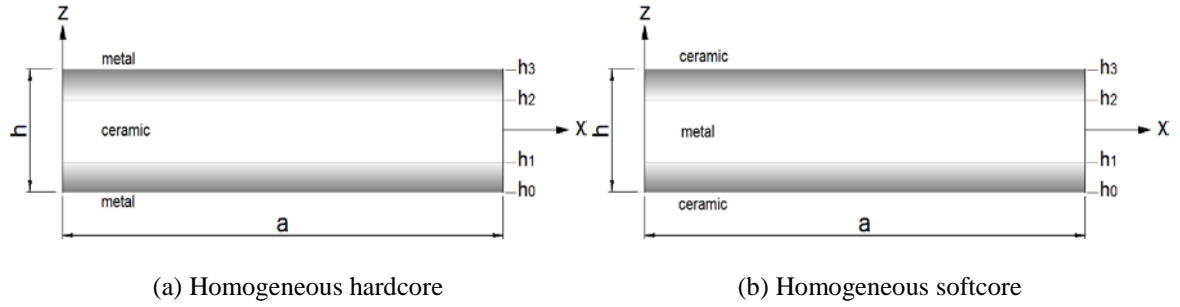


Fig. 1 Geometry of functionally graded sandwich plates

### 2.1 Kinematics and strains

The displacement field of the present study is expressed by:

$$\begin{aligned} u_1(x, y, z) &= u(x, y) - zw_{b,x} - \psi(z)w_{s,x} \\ u_2(x, y, z) &= v(x, y) - zw_{b,y} - \psi(z)w_{s,y} \\ u_3(x, y, z) &= w_b(x, y) + w_s(x, y) \end{aligned} \quad (1)$$

where the comma indicates partial differentiation with respect to the coordinate subscript that follows, the shape function  $\psi(z)$  at location  $z$  is given by (Grover *et al.* 2013):

$$\psi(z) = z \left( 1 + \frac{6}{h\sqrt{13}} \right) - \sinh^{-1} \left( \frac{3z}{h} \right) \quad (2)$$

and  $u, v, w_b$  and  $w_s$  are the four unknown displacements of the mid-plane of the plate. The in-plane and out-of-plane strains associated with the displacement field in Eq. (1) are obtained:

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_{xx}^{(0)} \\ \varepsilon_{yy}^{(0)} \\ \gamma_{xy}^{(0)} \end{Bmatrix} + z \begin{Bmatrix} \varepsilon_{xx}^{(1)} \\ \varepsilon_{yy}^{(1)} \\ \gamma_{xy}^{(1)} \end{Bmatrix} + \psi \begin{Bmatrix} \varepsilon_{xx}^{(2)} \\ \varepsilon_{yy}^{(2)} \\ \gamma_{xy}^{(2)} \end{Bmatrix} \quad (3a)$$

$$\begin{Bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} = g \begin{Bmatrix} \gamma_{xz}^{(0)} \\ \gamma_{yz}^{(0)} \end{Bmatrix} \quad (3b)$$

where  $g(z) = 1 - \psi_{,z}$ ;  $\varepsilon^{(0)}$  are membrane strains;  $\varepsilon^{(1)}, \varepsilon^{(2)}$  are curvatures and  $\gamma^{(0)}$  are transverse

shear strains. These strains are related to the displacements as follows:

$$\begin{Bmatrix} \varepsilon_{xx}^{(0)} \\ \varepsilon_{yy}^{(0)} \\ \gamma_{xy}^{(0)} \end{Bmatrix} = \begin{Bmatrix} u_{,x} \\ v_{,y} \\ u_{,y} + v_{,x} \end{Bmatrix}, \begin{Bmatrix} \varepsilon_{xx}^{(1)} \\ \varepsilon_{yy}^{(1)} \\ \gamma_{xy}^{(1)} \end{Bmatrix} = \begin{Bmatrix} -w_{b,xx} \\ -w_{b,yy} \\ -2w_{b,xy} \end{Bmatrix}, \begin{Bmatrix} \varepsilon_{xx}^{(2)} \\ \varepsilon_{yy}^{(2)} \\ \gamma_{xy}^{(2)} \end{Bmatrix} = \begin{Bmatrix} -w_{s,xx} \\ -w_{s,yy} \\ -2w_{s,xy} \end{Bmatrix} \quad (4a)$$

$$\begin{Bmatrix} \gamma_{xz}^{(0)} \\ \gamma_{yz}^{(0)} \end{Bmatrix} = \begin{Bmatrix} w_{s,x} \\ w_{s,y} \end{Bmatrix} \quad (4b)$$

Eqs. (3a) and (3b) can be rewritten in a compact form as:

$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}^{(0)} + z\boldsymbol{\varepsilon}^{(1)} + \psi\boldsymbol{\varepsilon}^{(2)} \quad (5a)$$

$$\boldsymbol{\gamma} = g\boldsymbol{\gamma}^{(0)} \quad (5b)$$

## 2.2 Equations of motion

Hamilton's principle is herein used to derive the equations of motion:

$$0 = \int_0^T (\delta U + \delta V - \delta K) dt \quad (6)$$

where  $\delta U$ ,  $\delta V$ , and  $\delta K$  are the variations of strain energy, work done, and kinetic energy, respectively.

The variation of strain energy is calculated by:

$$\begin{aligned} \delta U &= \int_A \sum_{j=1}^3 \int_{-h/2}^{h/2} \left( \sigma_{xx}^{(j)} \delta \varepsilon_{xx} + \sigma_{yy}^{(j)} \delta \varepsilon_{yy} + \sigma_{xy}^{(j)} \delta \gamma_{xy} + \sigma_{xz}^{(j)} \delta \gamma_{xz} + \sigma_{yz}^{(j)} \delta \gamma_{yz} \right) dA dz \\ &= \int_A \left[ N_{xx} \delta u_{,x} - M_{xx}^b \delta w_{b,xx} - M_{xx}^s \delta w_{s,xx} + N_{yy} \delta v_{,y} - M_{yy}^b \delta w_{b,yy} - M_{yy}^s \delta w_{s,yy} \right. \\ &\quad \left. + N_{xy} (\delta u_{,y} + \delta v_{,x}) - 2M_{xy}^b \delta w_{b,xy} - 2M_{xy}^s \delta w_{s,xy} + Q_x \delta w_{s,x} + Q_y \delta w_{s,y} \right] dA \end{aligned} \quad (7)$$

where  $N$ ,  $M$ , and  $Q$  are the stress resultants defined by:

$$(N_{xx}, N_{yy}, N_{xy}) = \sum_{j=1}^3 \int_{h_{j-1}}^{h_j} (\sigma_{xx}^{(j)}, \sigma_{yy}^{(j)}, \sigma_{xy}^{(j)}) dz \quad (8a)$$

$$(M_{xx}^b, M_{yy}^b, M_{xy}^b) = \sum_{j=1}^3 \int_{h_{j-1}}^{h_j} z (\sigma_{xx}^{(j)}, \sigma_{yy}^{(j)}, \sigma_{xy}^{(j)}) dz \quad (8b)$$

$$(M_{xx}^s, M_{yy}^s, M_{xy}^s) = \sum_{j=1}^3 \int_{h_{j-1}}^{h_j} \psi (\sigma_{xx}^{(j)}, \sigma_{yy}^{(j)}, \sigma_{xy}^{(j)}) dz \quad (8c)$$

$$(Q_x, Q_y) = \sum_{j=1}^3 \int_{h_{j-1}}^{h_j} g (\sigma_{xz}^{(j)}, \sigma_{yz}^{(j)}) dz \quad (8d)$$

The variation of work done by in-plane loads  $\bar{N}$  and transverse loads  $q$  is expressed by:

$$\delta V = - \int_A \bar{N} \delta w dA - \int_A q \delta w dA \quad (9)$$

where  $\bar{N} = N_{xx}^0 (w_{b,xx} + w_{s,xx}) + 2N_{xy}^0 (w_{b,xy} + w_{s,xy}) + N_{yy}^0 (w_{b,yy} + w_{s,yy})$ .

The variation of kinetic energy is determined by:

$$\begin{aligned} \delta K &= \int_V (\dot{w}_1 \delta \dot{w}_1 + \dot{w}_2 \delta \dot{w}_2 + \dot{w}_3 \delta \dot{w}_3) \rho(z) dA dz \\ &= \int_A I_0 [\dot{w}_b \delta \dot{w}_b + \dot{w}_s \delta \dot{w}_s + (\dot{w}_b + \dot{w}_s) \delta (\dot{w}_b + \dot{w}_s)] - I_1 (\dot{w}_b \delta \dot{w}_{b,x} + \dot{w}_{b,x} \delta \dot{w}_b + \dot{w}_s \delta \dot{w}_{s,y} + \dot{w}_{s,y} \delta \dot{w}_s) \\ &\quad + I_2 (\dot{w}_{b,x} \delta \dot{w}_{b,x} + \dot{w}_{b,y} \delta \dot{w}_{b,y}) - J_1 (\dot{w}_s \delta \dot{w}_{s,x} + \dot{w}_{s,x} \delta \dot{w}_s + \dot{w}_s \delta \dot{w}_{s,y} + \dot{w}_{s,y} \delta \dot{w}_s) \\ &\quad + K_2 (\dot{w}_{s,x} \delta \dot{w}_{s,x} + \dot{w}_{s,y} \delta \dot{w}_{s,y}) + J_2 (\dot{w}_{b,x} \delta \dot{w}_{s,x} + \dot{w}_{s,x} \delta \dot{w}_{b,x} + \dot{w}_{b,y} \delta \dot{w}_{s,y} + \dot{w}_{s,y} \delta \dot{w}_{b,y}) dA \end{aligned} \quad (10)$$

where the dot-superscript convention indicates the differentiation with respect to the time variable  $t$ ,  $\rho(z)$  is the mass density, and the inertia terms  $I_i, J_i, K_i$  are expressed by:

$$(I_0, I_1, I_2) = \sum_{j=1}^3 \int_{h_{j-1}}^{h_j} (1, z, z^2) \rho(z) dz \quad (11a)$$

$$(J_1, J_2, K_2) = \sum_{j=1}^3 \int_{h_{j-1}}^{h_j} (z, z^2, z^3) \rho(z) dz \quad (11b)$$

Substituting Eqs. (7), (9), and (10) into Eq. (6), integrating by parts, and collecting the coefficients  $\delta u, \delta v, \delta w_b, \delta w_s$ , the following equations of motion are obtained:

$$\delta u: N_{xx,x} + N_{xy,y} = I_0 \ddot{u} - I_1 \ddot{w}_{b,x} - J_1 \ddot{w}_{s,x} \quad (12a)$$

$$\delta v: N_{xy,x} + N_{yy,y} = I_0 \ddot{v} - I_1 \ddot{w}_{b,y} - J_1 \ddot{w}_{s,y} \quad (12b)$$

$$\begin{aligned} \delta w_b: M_{xx,xx}^b + 2M_{xy,xy}^b + M_{yy,yy}^b + \bar{N} + q \\ = I_0 (\ddot{w}_b + \ddot{w}_s) + I_1 (\ddot{w}_{b,x} + \ddot{w}_{s,y}) - I_2 (\ddot{w}_{b,xx} + \ddot{w}_{b,yy}) - J_2 (\ddot{w}_{s,xx} + \ddot{w}_{s,yy}) \end{aligned} \quad (12c)$$

$$\begin{aligned} \delta w_s: M_{xx,xx}^s + 2M_{xy,xy}^s + M_{yy,yy}^s + Q_{x,x} + Q_{y,y} + \bar{N} + q \\ = I_0 (\ddot{w}_b + \ddot{w}_s) + J_1 (\ddot{w}_{b,x} + \ddot{w}_{s,y}) - J_2 (\ddot{w}_{b,xx} + \ddot{w}_{b,yy}) - K_2 (\ddot{w}_{s,xx} + \ddot{w}_{s,yy}) \end{aligned} \quad (12d)$$

### 2.3 Constitutive equations

The effective material properties of FG sandwich plates according to the power-law form can be expressed by:

$$P^{(j)}(z) = (P_b - P_t)V_b^{(j)} + P_t \quad (13)$$

where  $P_t$  and  $P_b$  are the Young's moduli ( $E$ ), Poisson's ratio ( $\nu$ ), mass densities ( $\rho$ ) of materials located at the top and bottom surfaces, and at the core, respectively. The volume fraction function  $V_b^{(j)}$  defined by the power-law form as follows:

$$\begin{cases} V_b^{(1)} = \left( \frac{z - h_0}{h_1 - h_0} \right)^p & \text{for } z \in [h_0, h_1] \\ V_b^{(2)} = 1 & \text{for } z \in [h_1, h_2] \\ V_b^{(1)} = \left( \frac{z - h_3}{h_2 - h_3} \right)^p & \text{for } z \in [h_2, h_3] \end{cases} \quad (14)$$

where  $p$  is a power-law index, which is positive. Distribution of material with  $V_b$  through the plate thickness for the thickness ratio of layers (1-1-1) is displayed in Fig. 2. The linear constitutive relations of the  $j$ -th layer of FG sandwich plates are written as:

$$\begin{Bmatrix} \sigma_{xx}^{(j)} \\ \sigma_{yy}^{(j)} \\ \sigma_{xy}^{(j)} \end{Bmatrix} = \begin{bmatrix} C_{11}^{(j)} & C_{12}^{(j)} & 0 \\ C_{12}^{(j)} & C_{22}^{(j)} & 0 \\ 0 & 0 & C_{66}^{(j)} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx}^{(j)} \\ \varepsilon_{yy}^{(j)} \\ \gamma_{xy}^{(j)} \end{Bmatrix} \quad (15a)$$

$$\begin{Bmatrix} \sigma_{xz}^{(j)} \\ \sigma_{yz}^{(j)} \end{Bmatrix} = \begin{bmatrix} C_{55}^{(j)} & 0 \\ 0 & C_{44}^{(j)} \end{bmatrix} \begin{Bmatrix} \gamma_{xz}^{(j)} \\ \gamma_{yz}^{(j)} \end{Bmatrix} \quad (15b)$$

where

$$\begin{aligned} C_{11}^{(j)}(z) &= C_{22}^{(j)}(z) = \frac{E^{(j)}(z)}{1 - \nu^{(j)}(z)^2}, \quad C_{12}^{(j)}(z) = \nu^{(j)}(z) C_{11}^{(j)}(z) \\ C_{44}^{(j)}(z) &= C_{55}^{(j)}(z) = C_{66}^{(j)}(z) = \frac{E^{(j)}(z)}{2[1 + \nu^{(j)}(z)]} \end{aligned} \quad (16)$$

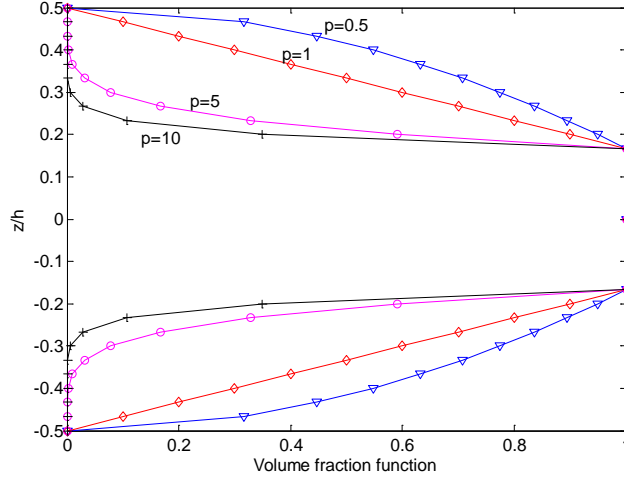


Fig. 2 Volume fraction function  $V_b$  with respect to the thickness ratio of layers (1-1-1)

Substituting Eq. (3a) into Eq. (15a) and the subsequent results into Eqs. (8a), (8b) and (8c), the stress resultants are obtained in terms of strains as following compact form:

$$\begin{Bmatrix} \mathbf{N} \\ \mathbf{M}^b \\ \mathbf{M}^s \end{Bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} & \mathbf{B}^s \\ \mathbf{B} & \mathbf{D} & \mathbf{D}^s \\ \mathbf{B}^s & \mathbf{D}^s & \mathbf{H}^s \end{bmatrix} \begin{Bmatrix} \boldsymbol{\varepsilon}^{(0)} \\ \boldsymbol{\varepsilon}^{(1)} \\ \boldsymbol{\varepsilon}^{(2)} \end{Bmatrix} \quad (17)$$

where  $\mathbf{A}, \mathbf{B}, \mathbf{D}, \mathbf{B}^s, \mathbf{D}^s, \mathbf{H}^s$  are the stiffnesses of the FG sandwich plate given by:

$$(\mathbf{A}, \mathbf{B}, \mathbf{D}, \mathbf{B}^s, \mathbf{D}^s, \mathbf{H}^s) = \sum_{j=1}^3 \int_{h_{j-1}}^{h_j} (1, z, z^2, \psi, z\psi, \psi^2) \mathbf{C}^{(j)}(z) dz \quad (18)$$

where  $\mathbf{C}^{(j)}(z)$  is reduced stiffness matrix of the  $j$ -th layer. Similarly, using Eqs. (3b), (15b) and (8d), the transverse shear forces can be calculated from the constitutive equations as:

$$\begin{Bmatrix} Q_x \\ Q_y \end{Bmatrix} = \begin{bmatrix} A_{55}^s & 0 \\ 0 & A_{44}^s \end{bmatrix} \begin{Bmatrix} \gamma_{xz}^{(0)} \\ \gamma_{yz}^{(0)} \end{Bmatrix} \quad (19)$$

or in a compact form as:

$$\mathbf{Q} = \mathbf{A}^s \boldsymbol{\gamma}^{(0)} \quad (20)$$

where the shear stiffnesses  $\mathbf{A}^s$  of the FG sandwich plate are defined by:



$$A_{44}^s = A_{55}^s = \sum_{j=1}^3 \int_{h_{j-1}}^{h_j} g^2 C_{44}^{(j)}(z) dz = \sum_{j=1}^3 \int_{h_{j-1}}^{h_j} g^2 C_{55}^{(j)}(z) dz \quad (21)$$

## 2.4 Equations of motion

Substituting Eqs. (17) and (20) into Eq. (12), the equations of motion can be expressed in terms of displacements ( $u, v, w_b, w_s$ ) as follow:

$$A_{11}u_{,xx} + A_{66}u_{,yy} + (A_{12} + A_{66})v_{,xy} - B_{11}w_{b,xxx} - (B_{12} + 2B_{66})w_{b,xyy} - B_{11}^s w_{s,xxx} - (B_{12}^s + 2B_{66}^s)w_{s,xyy} = I_0 \ddot{u} - I_1 \ddot{w}_{b,x} - J_1 \ddot{w}_{s,x} \quad (22a)$$

$$A_{22}v_{,yy} + A_{66}v_{,xx} + (A_{12} + A_{66})u_{,xy} - B_{22}w_{b,yyy} - (B_{12} + 2B_{66})w_{b,xyy} - B_{22}^s w_{s,yyy} - (B_{12}^s + 2B_{66}^s)w_{s,xyy} = I_0 \ddot{v} - I_1 \ddot{w}_{b,y} - J_1 \ddot{w}_{s,y} \quad (22b)$$

$$B_{11}u_{,xxx} + (B_{12} + 2B_{66})u_{,xyy} + (B_{12} + 2B_{66})v_{,xxy} + B_{22}v_{,yyy} - D_{11}w_{b,xxxx} - D_{22}w_{b,yyyy} - 2(D_{12} + 2D_{66})w_{b,xyyy} - D_{11}^s w_{s,xxxx} - D_{22}^s w_{s,yyyy} - 2(D_{12}^s + 2D_{66}^s)w_{s,xyyy} + \bar{N} + q = I_0 (\ddot{w}_b + \ddot{w}_s) + I_1 (\ddot{u}_{,x} + \ddot{v}_{,y}) - I_2 (\ddot{w}_{b,xx} + \ddot{w}_{b,yy}) - J_2 (\ddot{w}_{s,xx} + \ddot{w}_{s,yy}) \quad (22c)$$

$$B_{11}^s u_{,xxx} + (B_{11}^s + 2B_{66}^s)u_{,xyy} + (B_{12}^s + 2B_{66}^s)v_{,xxy} + B_{22}^s v_{,yyy} - D_{11}^s w_{b,xxxx} - D_{22}^s w_{b,yyyy} - 2(D_{12}^s + 2D_{66}^s)w_{b,xyyy} - H_{11}^s w_{s,xxxx} - H_{22}^s w_{s,yyyy} - 2(H_{12}^s + 2H_{66}^s)w_{s,xyyy} + A_{55}^s w_{s,xx} + A_{44}^s w_{s,yy} + \bar{N} + q = I_0 (\ddot{w}_b + \ddot{w}_s) + J_1 (\ddot{u}_{,x} + \ddot{v}_{,y}) - J_2 (\ddot{w}_{b,xx} + \ddot{w}_{b,yy}) - K_2 (\ddot{w}_{s,xx} + \ddot{w}_{s,yy}) \quad (22d)$$

## 2.5 Analytical solution for simply-supported FG sandwich plates

Consider a simply supported FG rectangular sandwich plate with length  $a$  and width  $b$  (Fig. 1). The Navier solution procedure is used to obtain the analytical solutions for the simply-supported boundary conditions, which are given by:

$$\begin{aligned} v = w_b = w_s = N_{xx} = M_{xx}^b = M_{xx}^s & \text{ on edges } x=0, a \\ u = w_b = w_s = N_{yy} = M_{yy}^b = M_{yy}^s & \text{ on edges } x=0, b \end{aligned} \quad (23)$$

The solution is assumed to be of the form:

$$u(x, y, t) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} U_{mn} \cos \lambda x \sin \mu y e^{i\omega t} \quad (24a)$$

$$v(x, y, t) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} V_{mn} \sin \lambda x \cos \mu y e^{i\omega t} \quad (24b)$$

$$w_b(x, y, t) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} W_{bmn} \sin \lambda x \sin \mu y e^{i\omega t} \quad (24c)$$

$$w_s(x, y, t) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} W_{smn} \sin \lambda x \sin \mu y e^{i\omega t} \quad (24c)$$

where  $\lambda = m\pi / a, \mu = n\pi / b, \omega$  is the frequency of free vibration of the plate,  $\sqrt{-1}$  the imaginary unit. The transverse load  $q$  is also expanded in the double-Fourier sine series as:

$$q(x, y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} Q_{mn} \sin \lambda x \sin \mu y \quad (25)$$

where  $Q_{mn} = q_0$  for sinusoidally distributed load and  $Q_{mn} = 16q_0 / mn\pi^2$  ( $m, n = 1, 3, 5, \dots$ ) for uniformly distributed load. Assuming that the plate is subjected to in-plane compressive loads of form:  $N_{xx}^0 = -N_0, N_{yy}^0 = -\gamma N_0$  (here  $\gamma$  is non-dimensional load parameter),  $N_{xy}^0 = 0$ . Substituting Eqs. (24) and (25) into Eq. (22), the following problem is obtained:

$$\begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} \\ k_{12} & k_{22} & k_{23} & k_{24} \\ k_{13} & k_{23} & k_{33} + \alpha & k_{34} + \alpha \\ k_{14} & k_{24} & k_{34} + \alpha & k_{44} + \alpha \end{bmatrix} - \omega^2 \begin{bmatrix} m_{11} & 0 & m_{13} & m_{14} \\ 0 & m_{22} & m_{23} & m_{24} \\ m_{13} & m_{23} & m_{33} & m_{34} \\ m_{14} & m_{24} & m_{34} & m_{44} \end{bmatrix} \begin{Bmatrix} U_{mn} \\ V_{mn} \\ W_{bmn} \\ W_{smn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ Q_{mn} \\ Q_{mn} \end{Bmatrix} \quad (26)$$

where

$$\begin{aligned} k_{11} &= A_{11}\lambda^2 + A_{66}\mu^2, k_{12} = (A_{12} + A_{66})\lambda\mu, k_{13} = -B_{11}\lambda^3 - (B_{12} + 2B_{66})\lambda\mu^2 \\ k_{14} &= -B_{11}\lambda^3 - (B_{12}^s + 2B_{66}^s)\lambda\mu^2, k_{22} = A_{66}\lambda^2 + A_{22}\mu^2, k_{23} = -B_{22}\mu^3 - (B_{12} + 2B_{66})\lambda^2\mu \\ k_{24} &= -B_{22}\mu^3 - (B_{12}^s + 2B_{66}^s)\lambda^2\mu, k_{33} = D_{11}\lambda^4 + 2(D_{12} + 2D_{66})\lambda^2\mu^2 + D_{22}\mu^4 \\ k_{34} &= D_{11}^s\lambda^4 + 2(D_{12}^s + 2D_{66}^s)\lambda^2\mu^2 + D_{22}^s\mu^4 \\ k_{44} &= H_{11}^s\lambda^4 + 2(H_{12}^s + 2H_{66}^s)\lambda^2\mu^2 + H_{22}^s\mu^4 + A_{55}^s\lambda^2 + A_{44}^s\mu^2 \\ m_{11} &= m_{22} = I_0, m_{13} = -\lambda I_1, m_{14} = -\lambda J_1, m_{23} = -\mu I_1, m_{24} = -\mu J_1 \\ m_{33} &= I_0 + I_2(\lambda^2 + \mu^2), m_{34} = I_0 + J_2(\lambda^2 + \mu^2), m_{44} = I_0 + K_2(\lambda^2 + \mu^2) \\ \alpha &= -N_0(\lambda^2 + \gamma\mu^2) \end{aligned} \quad (27)$$

Eq. (26) is a general form for bending, buckling and free vibration analysis of FG sandwich plates under in-plane and transverse loads. In order to solve bending problem, the in-plane compressive load  $N_0$  and mass matrix  $\mathbf{M}$  are set to zeros. The critical buckling loads ( $N_{cr}$ ) can be obtained from the stability problem  $|K_{ij}| = 0$  while the free vibration problem is achieved by omitting both in-plane and transverse loads.

## 2.6 Finite element formulation

Weak forms of Eqs. (12a)-(12d) on the element domain  $A_e$  can be obtained as follows:

$$\int_{A_e} \left[ N_{xx} \delta u_{,x} + N_{xy} \delta u_{,y} + (I_0 \ddot{u} - I_1 \ddot{w}_{b,x} - J_1 \ddot{w}_{s,x}) \delta u \right] dx dy = 0 \quad (28a)$$

$$\int_{A_e} \left[ N_{xy} \delta v_{,x} + N_{yy} \delta v_{,y} + (I_0 \ddot{v} - I_1 \ddot{w}_{b,y} - J_1 \ddot{w}_{s,y}) \delta v \right] dx dy = 0 \quad (28b)$$

$$\begin{aligned} \int_{A_e} & \left[ -M_{xx}^b \delta w_{b,xx} - 2M_{xy}^b \delta w_{b,xy} - M_{yy}^b \delta w_{b,yy} - q \delta w_b \right. \\ & + (N_{xx}^0 w_{b,x} + N_{xy}^0 w_{b,y}) \delta w_{b,x} + (N_{xy}^0 w_{b,x} + N_{yy}^0 w_{b,y}) \delta w_{b,y} \\ & \left. + I_0 (\ddot{w}_b + \ddot{w}_s) \delta w_b - (I_1 \ddot{u} - I_2 \ddot{w}_{b,x} - J_2 \ddot{w}_{s,x}) \delta w_{b,x} - (I_1 \ddot{v} - I_2 \ddot{w}_{b,y} - J_2 \ddot{w}_{s,y}) \delta w_{b,y} \right] dx dy \end{aligned} \quad (28c)$$

$$\begin{aligned} \int_{A_e} & \left[ -M_{xx}^s \delta w_{s,xx} - 2M_{xy}^s \delta w_{s,xy} - M_{yy}^s \delta w_{s,yy} + Q_x \delta w_{s,x} + Q_y \delta w_{s,y} - q \delta w_s \right. \\ & + (N_{xx}^0 w_{s,x} + N_{xy}^0 w_{s,y}) \delta w_{s,x} + (N_{xy}^0 w_{s,x} + N_{yy}^0 w_{s,y}) \delta w_{s,y} \\ & \left. + I_0 (\ddot{w}_b + \ddot{w}_s) \delta w_s - (J_1 \ddot{u} - J_2 \ddot{w}_{b,x} - K_2 \ddot{w}_{s,x}) \delta w_{s,x} - (J_1 \ddot{v} - J_2 \ddot{w}_{b,y} - K_2 \ddot{w}_{s,y}) \delta w_{s,y} \right] dx dy \end{aligned} \quad (28d)$$

A  $C^1$  four-node quadrilateral element with ten degrees of freedom per node is used. The displacements on each element are expressed as a combination of Lagrangian linear interpolation functions  $\psi_j$  at the  $j$ th node for  $u$  and  $v$  and Hermitian cubic interpolation functions  $\phi_j$  for  $w_b$  and  $w_s$  (see Thai and Choi 2013b):

$$u(x, y, t) = \sum_{j=1}^m \psi_j(x, y) u_j^e e^{i\omega t} \quad (29a)$$

$$v(x, y, t) = \sum_{j=1}^m \psi_j(x, y) v_j^e e^{i\omega t} \quad (29b)$$

$$w_b(x, y, t) = \sum_{j=1}^m \phi_j(x, y) \Delta_{bj}^e e^{i\omega t} \quad (29c)$$

$$w_s(x, y, t) = \sum_{j=1}^m \phi_j(x, y) \Delta_{sj}^e e^{i\omega t} \quad (29d)$$

where  $(u_j^e, v_j^e)$  denote the values of  $(u, v)$ ,  $(\Delta_{bj}^e, \Delta_{sj}^e)$  denote the values of  $(w_b, w_s)$  and their derivatives with respect to  $x$  and  $y$ , which are expressed by:

$$\Delta_b^e = \begin{bmatrix} w_b^e & w_{b,x}^e & w_{b,y}^e & w_{b,xy}^e \end{bmatrix}^T \quad (30a)$$

$$\Delta_s^e = \begin{bmatrix} w_s^e & w_{s,x}^e & w_{s,y}^e & w_{s,xy}^e \end{bmatrix}^T \quad (30b)$$

and the interpolation functions  $\psi_j$  and  $\phi_j$  for the  $j$ th node are given in terms of natural coordinates  $(\xi, \eta)$  as:

$$\psi_j = \frac{1}{4}(1 + \xi_j \xi)(1 - \eta_j \eta) \quad (31a)$$

$$\begin{aligned} \varphi_{j1} &= \frac{1}{16}(\xi + \xi_j)^2 (\xi \xi_j - 2)(\eta + \eta_j)^2 (\eta \eta_j - 2) \\ \varphi_{j2} &= -\frac{1}{16}\xi_j (\xi + \xi_j)^2 (1 - \xi \xi_j)(\eta + \eta_j)^2 (\eta \eta_j - 2) \\ \varphi_{j3} &= -\frac{1}{16}(\xi + \xi_j)^2 (\xi \xi_j - 2)\eta_j (\eta + \eta_j)^2 (\eta \eta_j - 1) \\ \varphi_{j4} &= \frac{1}{16}\xi_j (\xi + \xi_j)^2 (\xi \xi_j - 1)\eta_j (\eta + \eta_j)^2 (\eta \eta_j - 1) \end{aligned} \quad (31b)$$

Substituting Eqs. (29a)-(29d) into Eqs. (28a)-(28d) leads to:

$$\left( \begin{bmatrix} \mathbf{K}^{11} & \mathbf{K}^{12} & \mathbf{K}^{13} & \mathbf{K}^{14} \\ {}^T \mathbf{K}^{12} & \mathbf{K}^{22} & \mathbf{K}^{23} & \mathbf{K}^{24} \\ {}^T \mathbf{K}^{13} & {}^T \mathbf{K}^{23} & \mathbf{K}^{33} + \mathbf{G} & \mathbf{K}^{34} + \mathbf{G} \\ {}^T \mathbf{K}^{14} & {}^T \mathbf{K}^{24} & {}^T \mathbf{K}^{34} + \mathbf{G} & \mathbf{K}^{44} + \mathbf{G} \end{bmatrix} - \omega^2 \begin{bmatrix} \mathbf{M}^{11} & \mathbf{0} & \mathbf{M}^{13} & \mathbf{M}^{14} \\ \mathbf{0} & \mathbf{M}^{22} & \mathbf{M}^{23} & \mathbf{M}^{24} \\ {}^T \mathbf{M}^{13} & {}^T \mathbf{M}^{23} & \mathbf{M}^{33} & \mathbf{M}^{34} \\ {}^T \mathbf{M}^{14} & {}^T \mathbf{M}^{24} & {}^T \mathbf{M}^{34} & \mathbf{M}^{44} \end{bmatrix} \right) \begin{Bmatrix} \mathbf{u}^e \\ \mathbf{v}^e \\ \Delta_b^e \\ \Delta_s^e \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ \mathbf{F}^3 \\ \mathbf{F}^4 \end{Bmatrix} \quad (32)$$

where the coefficients of the stiffness matrix  $\mathbf{K}, \mathbf{G}$ , mass matrix  $\mathbf{M}$  and force vector  $\mathbf{F}$  are defined as follows:

$$K_{ij}^{11} = \int_{A_e} (A_{11} \psi_{i,x} \psi_{j,x} + A_{66} \psi_{i,y} \psi_{j,y}) dx dy \quad (33a)$$

$$K_{ij}^{12} = \int_{A_e} (A_{12} \psi_{i,x} \psi_{j,y} + A_{66} \psi_{i,y} \psi_{j,x}) dx dy \quad (33b)$$

$$K_{ij}^{13} = -\int_{A_e} [\psi_{i,x} (B_{11} \varphi_{j,xx} + B_{12} \varphi_{j,yy}) + 2\psi_{i,y} B_{66} \varphi_{j,xy}] dx dy \quad (33c)$$

$$K_{ij}^{14} = -\int_{A_e} [\psi_{i,x} (B_{11}^s \varphi_{j,xx} + B_{12}^s \varphi_{j,yy}) + 2\psi_{i,y} B_{66}^s \varphi_{j,xy}] dx dy \quad (33d)$$

$$K_{ij}^{22} = \int_{A_e} (A_{66} \psi_{i,x} \psi_{j,x} + A_{22} \psi_{i,y} \psi_{j,y}) dx dy \quad (33e)$$

$$K_{ij}^{23} = -\int_{A_e} [\psi_{i,y} (B_{22} \varphi_{j,yy} + B_{12} \varphi_{j,xx}) + 2\psi_{i,x} B_{66} \varphi_{j,xy}] dx dy \quad (33f)$$

$$K_{ij}^{24} = -\int_{A_e} [\psi_{i,y} (B_{22}^s \varphi_{j,yy} + B_{12}^s \varphi_{j,xx}) + 2\psi_{i,x} B_{66}^s \varphi_{j,xy}] dx dy \quad (33g)$$

$$K_{ij}^{33} = \int_{A_e} [D_{11} \varphi_{i,xx} \varphi_{j,xx} + D_{12} (\varphi_{i,xx} \varphi_{j,yy} + \varphi_{i,yy} \varphi_{j,xx}) + 4D_{66} \varphi_{i,xy} \varphi_{j,xy} + D_{22} \varphi_{i,yy} \varphi_{j,yy}] dx dy \quad (33h)$$

$$K_{ij}^{34} = \int_{A_e} [D_{11}^s \varphi_{i,xx} \varphi_{j,xx} + D_{12}^s (\varphi_{i,xx} \varphi_{j,yy} + \varphi_{i,yy} \varphi_{j,xx}) + 4D_{66}^s \varphi_{i,xy} \varphi_{j,xy} + D_{22}^s \varphi_{i,yy} \varphi_{j,yy}] dx dy \quad (33i)$$

$$K_{ij}^{44} = \int_{A_e} \left[ H_{11}^s \varphi_{i,xx} \varphi_{j,xx} + H_{12}^s (\varphi_{i,xx} \varphi_{j,yy} + \varphi_{i,yy} \varphi_{j,xx}) + H_{22}^s \varphi_{i,yy} \varphi_{j,yy} + 4H_{66}^s \varphi_{i,xy} \varphi_{j,xy} + A_{55}^s \varphi_{i,x} \varphi_{j,x} + A_{44}^s \varphi_{i,y} \varphi_{j,y} \right] dxdy \quad (33j)$$

$$M_{ij}^{11} = M_{ij}^{22} = \int_{A_e} I_0 \psi_i \psi_j dxdy \quad (33k)$$

$$M_{ij}^{13} = - \int_{A_e} I_1 \psi_i \varphi_{j,x} dxdy \quad (33l)$$

$$M_{ij}^{14} = - \int_{A_e} J_1 \psi_i \varphi_{j,x} dxdy \quad (33m)$$

$$M_{ij}^{23} = - \int_{A_e} I_1 \psi_i \varphi_{j,y} dxdy \quad (33n)$$

$$M_{ij}^{24} = - \int_{A_e} J_1 \psi_i \varphi_{j,y} dxdy \quad (33n)$$

$$M_{ij}^{33} = \int_{A_e} \left[ I_0 \varphi_i \varphi_j + I_2 (\varphi_{i,x} \varphi_{j,x} + \varphi_{i,y} \varphi_{j,y}) \right] dxdy \quad (33o)$$

$$M_{ij}^{34} = \int_{A_e} \left[ I_0 \varphi_i \varphi_j + J_2 (\varphi_{i,x} \varphi_{j,x} + \varphi_{i,y} \varphi_{j,y}) \right] dxdy \quad (33p)$$

$$M_{ij}^{44} = \int_{A_e} \left[ I_0 \varphi_i \varphi_j + K_2 (\varphi_{i,x} \varphi_{j,x} + \varphi_{i,y} \varphi_{j,y}) \right] dxdy \quad (33q)$$

$$F_{ij}^3 = F_{ij}^4 = \int_{A_e} q \varphi_i dxdy \quad (33r)$$

$$G_{ij} = \int_{A_e} \left[ N_{xx}^0 \varphi_{i,x} \varphi_{j,x} + N_{xy}^0 (\varphi_{i,x} \varphi_{j,y} + \varphi_{i,y} \varphi_{j,x}) + N_{yy}^0 \varphi_{i,y} \varphi_{j,y} \right] dxdy \quad (33s)$$

where the integrations in Eqs. (33) are numerically evaluated by Gauss quadrature integration rule.

### 3. Numerical results and discussion

In this section, a number of numerical examples are analyzed to verify the accuracy of present study and investigate the deflections, stresses, natural frequencies and critical buckling loads of FG sandwich plates. Two material combinations of metal and ceramic: Al/ZrO<sub>2</sub> and Al/Al<sub>2</sub>O<sub>3</sub> are considered. Their material properties are given in Table 1. Four types of boundary conditions are considered: simply supported edges (SSSS), clamped edges (CCCC), clamped-simply supported edges (CSCS: clamped at  $x=0,a$  and simply supported at  $y=0,b$ ) and clamped-free edges (CFCF: clamped at  $x=0,a$  and free at  $y=0,b$ ). Due to the symmetry, only quarter-plate model is used in the finite element modeling to reduce computational cost. A convergence study is carried out and the mesh size of 4×4 is sufficient to obtain an accurate solution. Unless mentioned otherwise, two cases of SSSS FG sandwich plates are considered:

- Hardcore: homogeneous core with Al<sub>2</sub>O<sub>3</sub> or ZrO<sub>2</sub> ( $E_b, \nu_b, \rho_b$ ) and FG faces with top and bottom surfaces made of Al ( $E_t, \nu_t, \rho_t$ )
- Softcore: homogeneous core with Al ( $E_b, \nu_b, \rho_b$ ) and FG faces with top and bottom surfaces made of Al<sub>2</sub>O<sub>3</sub> ( $E_t, \nu_t, \rho_t$ )

For convenience, the following non-dimensional parameters are used:

$$\begin{aligned}\bar{u}_3 &= \frac{10E_0h}{q_0a^2}u_3\left(\frac{a}{2}, \frac{b}{2}\right), \hat{u}_3 = \frac{100E_mh^3}{12(1-\nu_m^2)q_0a^4}u_3\left(\frac{a}{2}, \frac{b}{2}\right) \\ \bar{\sigma}_{xx}(z) &= \frac{10h^2}{q_0a^2}\sigma_{xx}\left(\frac{a}{2}, \frac{b}{2}, z\right), \bar{\sigma}_{xz}(z) = \frac{h}{q_0a}\sigma_{xz}\left(0, \frac{b}{2}, z\right) \\ \bar{N} &= \frac{N_{cr}a^2}{100E_0h^3}, \bar{\omega} = \omega \frac{a^2}{h} \sqrt{\rho_0 / E_0}, E_0 = 1 \text{ GPa}, \rho_0 = 1 \text{ kg/m}^3\end{aligned}\quad (34)$$

where  $E_m$  and  $\nu_m$  are the Young's modulus and Poisson's ratio of metal, respectively.

Table 1 Material properties of metal and ceramic

Materials	Young's modulus (GPa)	Mass density (kg/m3)	Poisson's ratio
Aluminum (Al)	70	2707	0.3
Zirconia (ZrO <sub>2</sub> )	151	3000	0.3
Zirconia (*ZrO <sub>2</sub> )	200	5700	0.3
Alumina (Al <sub>2</sub> O <sub>3</sub> )	380	3800	0.3

### 3.1 Results for bending analysis

For verification purpose, the center deflections, axial and transverse shear stresses of Al/ZrO<sub>2</sub> sandwich plates under sinusoidal loads are calculated in Tables 2-4. The present results are compared with those predicted by different shear deformation theories (FSDT, TSDT, SSDT and quasi-3D). It can be seen that the obtained results agree well with those reported by Zenkour (2005a, 2013), Neves *et al.* (2012c) and Bessaim *et al.* (2013), except some values of transverse shear stress. The present analytical solutions are better predictions with quasi-3D ones, which included both transverse shear and normal deformations, than TSDT and SSDT ones. The effects of the power-law index, thickness ratio of layers and side-to-thickness ratio on deflections, axial and shear stresses of Al/Al<sub>2</sub>O<sub>3</sub> sandwich plates with homogeneous hardcore and softcore are given in Tables 5-7. It can be seen that with the increase of power-law index, the deflections increase for sandwich plates with homogeneous hardcore, and decrease for ones with homogeneous softcore (Figure 3). The variations of axial and shear stresses through the thickness of (1-2-1) sandwich plate with homogeneous hardcore are plotted in Figure 4. The maximum axial stress is located inside the plate for  $p > 0$ . For example, with  $p = 10$  maximum axial stress is located at the interfaces of faces and core. Meanwhile, the maximum shear stress is located in the mid-plane of the plate (Figure 4b).

To further verify the accuracy of the present theory, Table 8 shows the comparison of the deflections of a thick Al/\*ZrO<sub>2</sub> CCCC square plate under uniform loads ( $b/h = 5$ ). It can be seen that the obtained finite element solutions are close to those generated by Gilhooley *et al.* (2007) using a quasi-3D theory, and those of Nguyen-Xuan *et al.* (2012) and Lee *et al.* (2009) using the FSDT. The effects of the boundary conditions, the power-law index and thickness ratio of layers on deflections, axial and shear stresses of Al/ZrO<sub>2</sub> sandwich plates with homogeneous hardcore are given in Tables 9-11 and the variation of nondimensional axial and transverse shear stresses

through the thickness of (2-2-1) Al/ZrO<sub>2</sub> CCCC sandwich plates is plotted in Fig. 5. It is seen that the deflections increase with an increase of the power-law index, lowest and highest ones correspond to the CCCC and CFCF cases, respectively.

### 3.2 Results for vibration and buckling analysis

To verify the accuracy of the present theory in predicting the vibration and buckling behavior of Al/Al<sub>2</sub>O<sub>3</sub> sandwich plates, their fundamental frequencies and critical buckling loads are calculated. Tables 12-15 contain the fundamental frequencies and critical buckling loads for six types of SSSS sandwich plates with different values of the power-law index. For buckling analysis, two types of in-plane loads: uniaxial compression ( $\gamma = 0$ ) and biaxial compressions ( $\gamma = 0.5, \gamma = 1$ ) are considered. It should be noted that the solutions reported by Li *et al.* (2008) were based on 3D linear theory of elasticity, whereas Zenkour (2005b), Meiche *et al.* (2011) and Bessaim *et al.* (2013) were based on a TSDT, SSDT and HSDT as well as quasi-3D. It is clear that the results of present study again agree well with previous solutions (Meiche *et al.* 2011, Li *et al.* 2008, Zenkour 2005b). Besides, it is found that for vibration, the present solutions are more in close agreement with those of Bessaim *et al.* (2013) and Li *et al.* (2008) than with those of Meiche *et al.* (2011) and Zenkour (2005b) in many cases. It implies that the proposed theory predicts more accurate than TSDT, SSDT and HSDT's model. Figures 6 and 7 show the fundamental frequencies and critical buckling loads of sandwich plates with respect to the power-law index. It can be seen from these figures that with the increase of the power-law index, they decrease for sandwich plates with homogeneous hardcore, and increase for ones with homogeneous softcore. Moreover, the effects of the boundary conditions on the fundamental frequency and critical buckling loads are also given in Tables 16 and 17. It is noticeable from Tables 5 and 12-17 and Figures 3, 6 and 7 that, for homogeneous hardcore, the lowest and highest values of deflection correspond to the (1-2-1) and (1-0-1) sandwich plates, and conversely for results of buckling load and natural frequency, respectively. It is due to the fact that these plates correspond to the highest and lowest volume fractions of the ceramic phase, and thus makes them become the hardest and softest ones.

Finally, the effects of the power-law index and side-to-thickness ratio on the first three natural frequencies and the critical buckling loads of (2-2-1) sandwich plate with homogeneous softcore is displayed in Figure 8. It is evident that they increase with an increase of the side-to-thickness ratio, and shear deformation effect becomes very effective in a relatively large region ( $b/h \leq 40$ ) (Figure 8). Three groups of curves are seen, for vibration analysis, the highest group is for the third mode ( $m = 1, n = 3$ ) and the lowest group is for the first mode ( $m = 1, n = 1$ ), for buckling analysis, the highest group is for the case of  $\gamma = 0$  (uniaxial compression) and the lowest group is for the case of  $\gamma = 1$  (biaxial compression).

## 4. Conclusions

A refined higher-order shear deformation theory for bending, vibration and buckling analysis of FG sandwich plates is proposed in this paper. It contains only four unknowns, accounts for a hyperbolic distribution of transverse shear stress and satisfies the traction free boundary conditions. Equations of motion are derived from Hamilton's principle. The Navier-type and finite element solutions are derived and compared with the existing solutions to verify the validity of the developed theory. Numerical results are obtained for FG sandwich plates with homogeneous hardcore and softcore to investigate the effects of the boundary conditions, power-law index,

thickness ratio of layers and side-to-thickness ratio on the deflections, stresses, critical buckling load and natural frequencies. It is observed that the present theory with four unknowns predicts the response with more accuracy and less computational cost as compared with five unknowns shear deformation theories.

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Table 2 Nondimensional center deflections  $\bar{u}_3$  of Al/ZrO<sub>2</sub> sandwich square plates with homogeneous hardcore (b/h=10).

p	Theory	1-0-1	2-1-2	2-1-1	1-1-1	2-2-1	1-2-1
0	Present (analytical)	0.19581	0.19581	0.19581	0.19581	0.19581	0.19581
	Present (FEM)	0.19706	0.19706	0.19706	0.19706	0.19706	0.19706
	Zenkour (2005a) (FSDT)	0.19607	0.19607	-	0.19607	0.19607	0.19607
	Zenkour (2005a) (TSDT)	0.19606	0.19606	-	0.19606	0.19606	0.19606
	Zenkour (2005a) (SSDT)	0.19605	0.19605	-	0.19605	0.19605	0.19605
	Zenkour (2013) (Quasi-3D)	0.19487	0.19487	-	0.19487	0.19487	0.19487
	Neves et al. (2012c) (Quasi-3D)	-	0.19490	0.19490	0.19490	0.19490	0.19490
	Bessaim et al. (2013) (Quasi-3D)	-	0.19486	0.19486	0.19486	0.19486	0.19486
1	Present	0.32300	0.30583	0.29637	0.29163	0.28054	0.27073
	Present (FEM)	0.32509	0.30782	0.29826	0.29352	0.28234	0.27248
	Zenkour (2005a) (FSDT)	0.32484	0.30750	-	0.29301	0.28168	0.27167
	Zenkour (2005a) (TSDT)	0.32358	0.30632		0.29199	0.28085	0.27094
	Zenkour (2005a) (SSDT)	0.32349	0.30624		0.29194	0.28082	0.27093
	Zenkour (2013) (Quasi-3D)	0.32001	0.30275		0.28867	0.27760	0.26815
	Neves et al. (2012c) (Quasi-3D)	-	0.30700	0.29750	0.29290	0.28200	0.27220
	Bessaim et al. (2013) (Quasi-3D)	-	0.30430	0.29448	0.29007	0.27874	0.26915
2	Present	0.37245	0.35158	0.33726	0.33237	0.31575	0.30237
	Present (FEM)	0.37487	0.35387	0.33940	0.33453	0.31776	0.30434
	Zenkour (2005a) (FSDT)	0.37514	0.35408	-	0.33441	0.31738	0.30370
	Zenkour (2005a) (TSDT)	0.37335	0.35231	-	0.33289	0.31617	0.30263
	Zenkour (2005a) (SSDT)	0.37319	0.35218	-	0.33280	0.31611	0.30260
	Zenkour (2013) (Quasi-3D)	0.36891	0.34737	-	0.32816	0.31152	0.29874
	Neves et al. (2012c) (Quasi-3D)	-	0.35190	0.33760	0.33290	0.31640	0.30320
	Bessaim et al. (2013) (Quasi-3D)	-	0.35001	0.33495	0.33068	0.31356	0.30060
5	Present	0.40799	0.39068	0.37234	0.37064	0.34900	0.33443
	Present (FEM)	0.41064	0.39323	0.37469	0.37306	0.35121	0.33661
	Zenkour (2005a) (FSDT)	0.41120	0.39418	-	0.37356	0.35123	0.33631
	Zenkour (2005a) (TSDT)	0.40927	0.39183	-	0.37145	0.34960	0.33480
	Zenkour (2005a) (SSDT)	0.40905	0.39160	-	0.37128	0.34950	0.33474
	Zenkour (2013) (Quasi-3D)	0.40532	0.38612	-	0.36546	0.34361	0.32966
	Neves et al. (2012c) (Quasi-3D)	-	0.39050	0.37220	0.37050	0.34900	0.33470
	Bessaim et al. (2013) (Quasi-3D)	-	0.38934	0.36981	0.36902	0.34649	0.33255
10	Present	0.41645	0.40270	0.38363	0.38452	0.36142	0.34777
	Present (FEM)	0.41913	0.40533	0.38605	0.38703	0.36370	0.35004
	Zenkour (2005a) (FSDT)	0.41919	0.40657	-	0.38787	0.36395	0.34996
	Zenkour (2005a) (TSDT)	0.41772	0.40407	-	0.38551	0.36215	0.34824
	Zenkour (2005a) (SSDT)	0.41750	0.40376	-	0.38490	0.34916	0.34119

Zenkour (2013) (Quasi-3D)	0.41448	0.39856	-	0.37924	0.35577	0.34259
Neves et al. (2012c) (Quasi-3D)	-	0.40260	0.38350	0.38430	0.36120	0.34800
Bessaim et al. (2013) (Quasi-3D)	-	0.40153	0.38111	0.38303	0.35885	0.34591

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Table 3 Nondimensional axial stress  $\bar{\sigma}_{xx}(h/2)$  of Al/ZrO<sub>2</sub> sandwich square plates with homogeneous hardcore (b/h=10).

p	Theory	1-0-1	2-1-2	2-1-1	1-1-1	2-2-1	1-2-1
0	Present	1.99816	1.99816	1.99816	1.99816	1.99816	1.99816
	Present (FEM)	2.1514	2.1514	2.1514	2.1514	2.1514	2.1514
	Zenkour (2005a) (FSDT)	1.97576	1.97576	-	1.97576	1.97576	1.97576
	Zenkour (2005a) (TSDT)	2.04985	2.04985	-	2.04985	2.04985	2.04985
	Zenkour (2005a) (SSDT)	2.05452	2.05452	-	2.05452	2.05452	2.05452
	Zenkour (2013) (Quasi-3D)	2.00773	2.00773	-	2.00773	2.00773	2.00773
	Neves et al. (2012c) (Quasi-3D)	-	2.00660	2.00640	2.00660	2.00650	2.00640
	Bessaim et al. (2013) (Quasi-3D)	-	1.99524	1.99524	1.99524	1.99524	1.99524
1	Present	1.54644	1.46498	1.35921	1.39614	1.29063	1.29390
	Present (FEM)	1.6334	1.5461	1.4418	1.4749	1.3699	1.3705
	Zenkour (2005a) (FSDT)	1.53245	1.45167	-	1.38303	1.27749	1.28096
	Zenkour (2005a) (TSDT)	1.57923	1.49587	-	1.42617	1.32062	1.32309
	Zenkour (2005a) (SSDT)	1.58204	1.49859	-	1.42892	1.32342	1.32590
	Zenkour (2013) (Quasi-3D)	1.57004	1.48833	-	1.41781	1.30907	1.31204
	Neves et al. (2012c) (Quasi-3D)	-	1.48130	1.37680	1.41370	1.30920	1.31330
	Bessaim et al. (2013) (Quasi-3D)	-	1.46131	1.35053	1.39243	1.28274	1.29030
2	Present	1.78579	1.68875	1.53218	1.59602	1.43915	1.44937
	Present (FEM)	1.8818	1.7745	1.6200	1.6778	1.5217	1.5280
	Zenkour (2005a) (FSDT)	1.77085	1.67496	-	1.58242	1.42528	1.43580
	Zenkour (2005a) (TSDT)	1.82167	1.72144	-	1.62748	1.47095	1.47988
	Zenkour (2005a) (SSDT)	1.82450	1.72412	-	1.63025	1.47387	1.48283
	Zenkour (2013) (Quasi-3D)	1.81509	1.72030	-	1.62591	1.46372	1.47421
	Neves et al. (2012c) (Quasi-3D)	-	1.69940	1.54560	1.60880	1.45430	1.46590
	Bessaim et al. (2013) (Quasi-3D)	-	1.68472	1.52101	1.59170	1.42887	1.44497
5	Present	1.95216	1.87869	1.68134	1.78347	1.57842	1.60691
	Present (FEM)	2.0641	1.9704	1.7762	1.8686	1.6646	1.6875
	Zenkour (2005a) (FSDT)	1.93576	1.86479	-	1.76988	1.56401	1.59309
	Zenkour (2005a) (TSDT)	1.99272	1.91302	-	1.81580	1.61181	1.63814
	Zenkour (2005a) (SSDT)	1.99567	1.91547	-	1.81838	1.61477	1.64106
	Zenkour (2013) (Quasi-3D)	1.97912	1.91504	-	1.82018	1.60953	1.63906
	Neves et al. (2012c) (Quasi-3D)	-	1.88380	1.69090	1.79060	1.58930	1.61950
	Bessaim et al. (2013) (Quasi-3D)	-	1.87516	1.66856	1.77919	1.56627	1.60203
10	Present	1.98591	1.93573	1.73141	1.85102	1.63059	1.67244
	Present (FEM)	2.1111	2.0316	1.8309	1.9381	1.7189	1.7539
	Zenkour (2005a) (FSDT)	1.96780	1.92165	-	1.83754	1.61645	1.65844
	Zenkour (2005a) (TSDT)	2.03036	1.97126	-	1.88376	1.66660	1.70417
	Zenkour (2005a) (SSDT)	2.03360	1.97313	-	1.88147	1.61979	1.64851

Zenkour (2013) (Quasi-3D)	2.00692	1.97075	-	1.89162	2.18558	1.67350
Neves et al. (2012c) (Quasi-3D)	-	1.93970	1.74050	1.85590	1.63950	1.68320
Bessaim et al. (2013) (Quasi-3D)	-	1.93266	1.71835	1.84705	1.61792	1.66754

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Table 4 Nondimensional transverse shear stress  $\bar{\sigma}_{xz}(0)$  of Al/ZrO<sub>2</sub> sandwich square plates with homogeneous hardcore (b/h=10).

p	Theory	1-0-1	2-1-2	2-1-1	1-1-1	2-2-1	1-2-1
0	Present	0.27209	0.27209	0.27209	0.27209	0.27209	0.27209
	Present (FEM)	0.2702	0.2702	0.2702	0.2702	0.2702	0.2702
	Zenkour (2005a) (FSDT)	0.19099	0.19099	-	0.19099	0.19099	0.19099
	Zenkour (2005a) (TSDT)	0.23857	0.23857	-	0.23857	0.23857	0.23857
	Zenkour (2005a) (SSDT)	0.24618	0.24618	-	0.24618	0.24618	0.24618
	Zenkour (2013) (Quasi-3D)	0.23910	0.23910	-	0.23910	0.23910	0.23910
	Neves et al. (2012c) (Quasi-3D)	-	0.25380	0.22910	0.24610	0.24110	0.23630
	Bessaim et al. (2013) (Quasi-3D)	-	0.23794	0.23794	0.23794	0.23794	0.23794
1	Present	0.32334	0.30125	0.30369	0.29273	0.29237	0.28656
	Present (FEM)	0.3211	0.29916	0.30158	0.29070	0.29034	0.28457
	Zenkour (2005a) (FSDT)	0.26099	0.24316	-	0.23257	0.22762	0.22057
	Zenkour (2005a) (TSDT)	0.29203	0.27104	-	0.26117	0.25951	0.25258
	Zenkour (2005a) (SSDT)	0.29907	0.27774	-	0.26809	0.26680	0.26004
	Zenkour (2013) (Quasi-3D)	0.36531	0.34366	-	0.32853	0.31785	0.30845
	Neves et al. (2012c) (Quasi-3D)	-	0.27450	0.26400	0.26430	0.25940	0.24960
	Bessaim et al. (2013) (Quasi-3D)	-	0.27050	0.27017	0.26060	0.25890	0.25196
2	Present	0.35524	0.31480	0.31961	0.30063	0.30066	0.29132
	Present (FEM)	0.35278	0.31262	0.31738	0.29855	0.29858	0.28930
	Zenkour (2005a) (FSDT)	0.29731	0.26752	-	0.25077	0.24316	0.23257
	Zenkour (2005a) (TSDT)	0.32622	0.28838	-	0.27188	0.26939	0.25834
	Zenkour (2005a) (SSDT)	0.33285	0.29422	-	0.27807	0.27627	0.26543
	Zenkour (2013) (Quasi-3D)	0.41778	0.38601	-	0.36417	0.34824	0.33543
	Neves et al. (2012c) (Quasi-3D)	-	0.27600	0.28770	0.26680	0.26360	0.25230
	Bessaim et al. (2013) (Quasi-3D)	-	0.28792	0.28742	0.27138	0.26885	0.25776
5	Present	0.41674	0.33526	0.34435	0.31020	0.31131	0.29569
	Present (FEM)	0.41386	0.33293	0.34195	0.30805	0.30914	0.29364
	Zenkour (2005a) (FSDT)	0.34538	0.29731	-	0.27206	0.26099	0.24596
	Zenkour (2005a) (TSDT)	0.38634	0.31454	-	0.28643	0.28265	0.26512
	Zenkour (2005a) (SSDT)	0.39370	0.31930	-	0.29150	0.28895	0.27153
	Zenkour (2013) (Quasi-3D)	0.46890	0.42723	-	0.39918	0.37791	0.36234
	Neves et al. (2012c) (Quasi-3D)	-	0.27120	0.33770	0.26550	0.26690	0.25460
	Bessaim et al. (2013) (Quasi-3D)	-	0.31419	0.31293	0.28606	0.28217	0.26463
10	Present	0.47099	0.35092	0.36250	0.31657	0.31837	0.29777
	Present (FEM)	0.46770	0.34846	0.35995	0.31436	0.31614	0.2957
	Zenkour (2005a) (FSDT)	0.37277	0.31316	-	0.28299	0.26998	0.25257
	Zenkour (2005a) (TSDT)	0.43206	0.33242	-	0.29566	0.29080	0.26895
	Zenkour (2005a) (SSDT)	0.44147	0.33644	-	0.29529	0.29671	0.27676

Zenkour (2013) (Quasi-3D)	0.49051	0.44435	-	0.41385	0.39045	0.37390
Neves et al. (2012c) (Quasi-3D)	-	0.26710	0.38060	0.26390	0.26920	0.25680
Bessaim et al. (2013) (Quasi-3D)	-	0.33210	0.32959	0.29534	0.29036	0.26850

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Table 5 Nondimensional center deflections  $\bar{u}_3$  of Al/Al<sub>2</sub>O<sub>3</sub> sandwich square plates with homogeneous hardcore and softcore.

Core	b/h	p	1-0-1	2-1-2	2-1-1	1-1-1	2-2-1	1-2-1
Hardcore	5	0	0.02248	0.02248	0.02248	0.02248	0.02248	0.02248
		0.5	0.03816	0.03543	0.03434	0.03346	0.03217	0.03084
		1	0.05327	0.04720	0.04476	0.04286	0.04009	0.03730
		5	0.10161	0.08770	0.07916	0.07489	0.06573	0.05748
		10	0.10996	0.09685	0.08701	0.08346	0.07255	0.06321
	10	0	0.07781	0.07781	0.07781	0.07781	0.07781	0.07781
		0.5	0.13759	0.12758	0.12319	0.12009	0.11500	0.10993
		1	0.19579	0.17329	0.16343	0.15660	0.14569	0.13494
		5	0.37990	0.33228	0.29728	0.28268	0.24614	0.21394
		10	0.40569	0.36793	0.32717	0.31662	0.27294	0.23663
Softcore	5	0	0.12201	0.12201	0.12201	0.12201	0.12201	0.12201
		0.5	0.04165	0.04658	0.04823	0.05035	0.05315	0.05587
		1	0.03285	0.03762	0.03885	0.04145	0.04384	0.04686
		5	0.02437	0.02797	0.02868	0.03163	0.03331	0.03732
		10	0.02337	0.02653	0.02720	0.03002	0.03158	0.03580
	10	0	0.42238	0.42238	0.42238	0.42238	0.42238	0.42238
		0.5	0.12984	0.14143	0.14975	0.15187	0.16356	0.17012
		1	0.10240	0.11192	0.11900	0.12083	0.13119	0.13655
		5	0.08054	0.08577	0.09074	0.09198	0.09976	0.10404
		10	0.07887	0.08302	0.08752	0.08852	0.09575	0.09985

Table 6 Nondimensional axial stress  $\bar{\sigma}_{xx}(h/2)$  of Al/Al<sub>2</sub>O<sub>3</sub> sandwich square plates with homogeneous hardcore and softcore.

Core	b/h	p	1-0-1	2-1-2	2-1-1	1-1-1	2-2-1	1-2-1
Hardcore	5	0	2.06499	2.06499	2.06499	2.06499	2.06499	2.06499
		0.5	0.67811	0.62884	0.57977	0.59191	0.54395	0.54163
		1	0.96663	0.85597	0.75027	0.77382	0.67469	0.66676
		5	1.87021	1.63974	1.29282	1.39738	1.08565	1.05967
		10	1.99309	1.81363	1.41179	1.56386	1.19180	1.17174
	10	0	1.99816	1.99816	1.99816	1.99816	1.99816	1.99816
		0.5	0.66015	0.61190	0.56273	0.57540	0.52770	0.52564
		1	0.94491	0.83618	0.73015	0.75472	0.65599	0.64846
		5	1.84076	1.61688	1.26621	1.37470	1.06237	1.03699
		10	1.95752	1.79109	1.38334	1.54156	1.16787	1.14867
Softcore	5	0	2.06499	2.06499	2.06499	2.06499	2.06499	2.06499
		0.5	3.37291	3.64881	4.19439	3.90261	4.58333	4.36193
		1	2.66925	2.88726	3.34333	3.09373	3.68874	3.47176
		5	2.12775	2.24453	2.54168	2.38099	2.80713	2.64695
		10	2.08876	2.18124	2.44351	2.30207	2.69319	2.54919
	10	0	1.99816	1.99816	1.99816	1.99816	1.99816	1.99816
		0.5	3.20390	3.45074	4.02743	3.69235	4.39708	4.14831
		1	2.52909	2.71004	3.20399	2.89735	3.52620	3.26777
		5	2.03584	2.10819	2.44190	2.20993	2.67847	2.44621
		10	2.00839	2.05744	2.35233	2.14077	2.57315	2.35050

Table 7 Nondimensional transverse shear stress  $\bar{\sigma}_{xz}(0)$  of Al/Al<sub>2</sub>O<sub>3</sub> sandwich square plates with homogeneous hardcore and softcore.

Core	b/h	p	1-0-1	2-1-2	2-1-1	1-1-1	2-2-1	1-2-1
Hardcore	5	0	0.27096	0.27096	0.27096	0.27096	0.27096	0.27096
		0.5	0.32178	0.30204	0.30354	0.29395	0.29299	0.28747
		1	0.36388	0.32400	0.32711	0.30908	0.30742	0.29773
		5	0.58073	0.39103	0.40705	0.34678	0.34575	0.32084
		10	0.76034	0.42000	0.44374	0.35760	0.35821	0.32576
	10	0	0.27209	0.27209	0.27209	0.27209	0.27209	0.27209
		0.5	0.32265	0.30287	0.30441	0.29479	0.29385	0.28834
		1	0.36473	0.32475	0.32790	0.30983	0.30818	0.29848
		5	0.58197	0.39178	0.40786	0.34747	0.34642	0.32148
		10	0.76213	0.42075	0.44461	0.35830	0.35889	0.32641
Softcore	5	0	0.27096	0.27096	0.27096	0.27096	0.27096	0.27096
		0.5	0.15838	0.19508	0.18888	0.21699	0.21588	0.23615
		1	0.12365	0.16621	0.15851	0.19594	0.19422	0.22528
		5	0.07031	0.10933	0.10218	0.14659	0.14442	0.19659
		10	0.06038	0.09617	0.09006	0.13304	0.13106	0.18704
	10	0	0.27209	0.27209	0.27209	0.27209	0.27209	0.27209
		0.5	0.15951	0.19657	0.19019	0.21861	0.21742	0.23779
		1	0.12450	0.16751	0.15959	0.19749	0.19565	0.22694
		5	0.07070	0.11015	0.10278	0.14781	0.14545	0.19819
		10	0.06068	0.09685	0.09054	0.13414	0.13197	0.18860

Table 8 Nondimensional center deflections  $\hat{u}_3$  of Al/\*ZrO<sub>2</sub> CCCC square plates under uniform load (b/h=5).

Theory	p			
	0	0.5	1	2
Present (FEM)	0.0745	0.0992	0.1156	0.1354
Gilhooley <i>et al.</i> (2007) (Quasi-3D)	0.0731	0.1073	0.1253	0.1444
Nguyen-Xuan <i>et al.</i> (2012) (FSDT)	0.0788	0.1051	0.1227	0.1420
Lee <i>et al.</i> (2009) (FSDT)	0.0774	0.1034	0.1207	0.1404

Table 9 Nondimensional center deflections  $\bar{u}_3$  of Al/ZrO<sub>2</sub> sandwich square plates with homogeneous hardcore and various boundary conditions (b/h=10).

Boundary conditions	p	1-0-1	2-1-2	2-1-1	1-1-1	2-2-1	1-2-1
CCCC	0	0.1128	0.1128	0.1128	0.1128	0.1128	0.1128
	0.5	0.1557	0.1495	0.1467	0.1448	0.1414	0.1381
	1	0.1824	0.1726	0.1675	0.1648	0.1588	0.1535
	5	0.2304	0.2187	0.2091	0.2073	0.1957	0.1878
	10	0.2365	0.2255	0.2156	0.2149	0.2024	0.1949
CSCS	0	0.1601	0.1601	0.1601	0.1601	0.1601	0.1601
	0.5	0.2225	0.2137	0.2095	0.2069	0.2018	0.1970
	1	0.2614	0.2474	0.2399	0.2361	0.2273	0.2195
	5	0.3302	0.3147	0.3003	0.2985	0.2813	0.2698
	10	0.3308	0.3245	0.3096	0.3095	0.2911	0.2803
CFCF	0	0.2238	0.2238	0.2238	0.2238	0.2238	0.2238
	0.5	0.3099	0.2977	0.2919	0.2882	0.2812	0.2747
	1	0.3636	0.3441	0.3338	0.3285	0.3163	0.3057
	5	0.4593	0.4369	0.4172	0.4143	0.3906	0.3748
	10	0.4709	0.4505	0.4302	0.4295	0.4042	0.3893

Table 10 Nondimensional axial stress  $\bar{\sigma}_{xx}(h/2)$  of Al/ZrO<sub>2</sub> sandwich square plates with homogeneous hardcore and various boundary conditions (b/h=10).

Boundary conditions	p	1-0-1	2-1-2	2-1-1	1-1-1	2-2-1	1-2-1
CCCC	0	1.6045	1.6045	1.6045	1.6045	1.6045	1.6045
	0.5	1.0350	0.9942	0.9513	0.9626	0.9189	0.9176
	1	1.2142	1.1489	1.0749	1.0966	1.0216	1.0206
	5	1.5345	1.4588	1.3215	1.3828	1.2374	1.2507
	10	1.5739	1.5044	1.3630	1.4335	1.2774	1.2988
CSCS	0	2.2066	2.2066	2.2066	2.2066	2.2066	2.2066
	0.5	1.4239	1.3677	1.3069	1.3236	1.2618	1.2603
	1	1.6747	1.5852	1.4801	1.5121	1.4062	1.4052
	5	2.1162	2.0199	1.8244	1.9155	1.7096	1.7300
	10	2.1646	2.0826	1.8807	1.9867	1.7655	1.7980
CFCF	0	2.7960	2.7960	2.7960	2.7960	2.7960	2.7960
	0.5	1.7988	1.7278	1.6531	1.6725	1.5963	1.5933
	1	2.1121	1.9999	1.8709	1.9082	1.7776	1.7745
	5	2.6704	2.5443	2.3050	2.4123	2.1588	2.1801
	10	2.7349	2.6235	2.3769	2.5015	2.2293	2.2650

Table 11 Nondimensional transverse shear stress  $\bar{\sigma}_{xz}(0)$  of Al/ZrO<sub>2</sub> sandwich square plates with homogeneous hardcore and various boundary conditions (b/h=10).

Boundary conditions	p	1-0-1	2-1-2	2-1-1	1-1-1	2-2-1	1-2-1
CCCC	0	0.4235	0.4235	0.4235	0.4235	0.4235	0.4235
	0.5	0.4734	0.4550	0.4568	0.4473	0.4466	0.4411
	1	0.5079	0.4733	0.4776	0.4598	0.4594	0.4498
	5	0.6544	0.5279	0.5452	0.4882	0.4923	0.4649
	10	0.7381	0.5528	0.5741	0.4984	0.5044	0.4682
CSCS	0	0.4319	0.4319	0.4319	0.4319	0.4319	0.4319
	0.5	0.4825	0.4638	0.4657	0.4559	0.4553	0.4497
	1	0.5176	0.4824	0.4870	0.4686	0.4683	0.4584
	5	0.6670	0.5380	0.5569	0.4974	0.5028	0.4738
	10	0.7523	0.5633	0.5867	0.5079	0.5151	0.4771
CFCF	0	0.6424	0.6424	0.6424	0.6424	0.6424	0.6424
	0.5	0.7181	0.6902	0.6930	0.6784	0.6775	0.6691
	1	0.7704	0.7180	0.7247	0.6974	0.6969	0.6823
	5	0.9926	0.8008	0.8282	0.7404	0.7475	0.7052
	10	1.1195	0.8385	0.8723	0.7560	0.7661	0.7101

Table 12 Nondimensional fundamental frequency  $\bar{\omega}$  of Al/Al<sub>2</sub>O<sub>3</sub> sandwich square plates with homogeneous hardcore (b/h=10).

p	Theory	1-0-1	2-1-2	2-1-1	1-1-1	2-2-1	1-2-1
0	Present	1.82562	1.82562	1.82562	1.82562	1.82562	1.82562
	Present (FEM)	1.8219	1.8219	1.8219	1.8219	1.8219	1.8219
	Zenkour (2005b) (TSDT)	1.82445	1.82445	1.82445	1.82445	1.82445	1.82445
	Zenkour (2005b) (SSDT)	1.82452	1.82452	1.82452	1.82452	1.82452	1.82452
	Meiche et al. (2011) (HSDT)	1.82449	1.82449	1.82449	1.82449	1.82449	1.82449
	Bessaim et al. (2013) (Quasi-3D)	1.82682	1.82682	-	1.82682	1.82682	1.82682
	Li et al. (2008) (3D)	1.82682	1.82682	-	1.82682	1.82682	1.82682
0.5	Present	1.44417	1.48415	1.50640	1.51927	1.54717	1.57453
	Present (FEM)	1.4394	1.4797	1.5021	1.5151	1.5431	1.5705
	Zenkour (2005b) (TSDT)	1.44424	1.48408	1.51253	1.51922	1.55199	1.57451
	Zenkour (2005b) (SSDT)	1.44436	1.48418	1.51258	1.51927	1.55202	1.57450
	Meiche et al. (2011) (HSDT)	1.44419	1.48405	1.50636	1.51922	1.54714	1.57458
	Bessaim et al. (2013) (Quasi-3D)	1.44621	1.48611	-	1.52130	1.55016	1.57670
	Li et al. (2008) (3D)	1.44614	1.48608	-	1.52131	1.54926	1.57668
1	Present	1.24410	1.30086	1.33397	1.35385	1.39612	1.43954
	Present (FEM)	1.2420	1.2985	1.3317	1.3513	1.3937	1.4368
	Zenkour (2005b) (TSDT)	1.24320	1.30011	1.34888	1.35333	1.40789	1.43934
	Zenkour (2005b) (SSDT)	1.24335	1.30023	1.34894	1.35339	1.40792	1.43931
	Meiche et al. (2011) (HSDT)	1.24310	1.30004	1.33328	1.35331	1.39559	1.43940
	Bessaim et al. (2013) (Quasi-3D)	1.24495	1.30195	-	1.35527	1.39987	1.44143
	Li et al. (2008) (3D)	1.24470	1.30181	-	1.35523	1.39763	1.44137
5	Present	0.94751	0.98289	1.03129	1.04533	1.10952	1.17412
	Present (FEM)	0.94618	0.98133	1.0301	1.0463	1.1081	1.1720
	Zenkour (2005b) (TSDT)	0.94598	0.98184	1.07432	1.04466	1.14731	1.17397
	Zenkour (2005b) (SSDT)	0.94630	0.98207	1.07445	1.04481	1.14741	1.17399
	Meiche et al. (2011) (HSDT)	0.94574	0.98166	1.03033	1.04455	1.10875	1.17397
	Bessaim et al. (2013) (Quasi-3D)	0.94716	0.98311	-	1.04613	1.11723	1.17579
	Li et al. (2008) (3D)	0.94476	0.98103	-	1.04532	1.10983	1.17567
10	Present	0.93024	0.94422	0.99288	0.99632	1.06172	1.12338
	Present (FEM)	0.92904	0.94278	0.99188	0.99467	1.0605	1.1214
	Zenkour (2005b) (TSDT)	0.92839	0.94297	1.03862	0.99551	1.10533	1.12314
	Zenkour (2005b) (SSDT)	0.92875	0.94332	1.04558	0.99519	1.04154	1.13460
	Meiche et al. (2011) (HSDT)	0.92811	0.94275	0.99184	0.99536	1.06081	1.12311
	Bessaim et al. (2013) (Quasi-3D)	0.92952	0.94410	-	0.99684	1.07015	1.12486
	Li et al. (2008) (3D)	0.92727	0.94078	-	0.99523	1.06104	1.12466



Table 13 Nondimensional fundamental frequency  $\bar{\omega}$  of Al/Al<sub>2</sub>O<sub>3</sub> sandwich square plates with homogeneous softcore (b/h=10).

p	Theory	1-0-1	2-1-2	2-1-1	1-1-1	2-2-1	1-2-1
0	Present	0.92836	0.92836	0.92836	0.92836	0.92836	0.92836
	Present (FEM)	0.9273	0.9273	0.9273	0.9273	0.9273	0.9273
	Bessaim et al. (2013) (Quasi-3D)	0.92897	0.92897	-	0.92897	0.92897	0.92897
	Li et al. (2008) (3D)	0.92897	0.92897	-	0.92897	0.92897	0.92897
0.5	Present	1.57228	1.52489	1.48614	1.48373	1.43528	1.41650
	Present (FEM)	1.5719	1.5243	1.4857	1.4831	1.4347	1.4158
	Bessaim et al. (2013) (Quasi-3D)	1.57705	1.53096	-	1.48853	1.44040	1.41788
	Li et al. (2008) (3D)	1.57352	1.52588	-	1.48459	1.43419	1.41662
1	Present	1.72003	1.67404	1.63014	1.63030	1.57356	1.55693
	Present (FEM)	1.7173	1.6716	1.6280	1.6281	1.5717	1.5549
	Bessaim et al. (2013) (Quasi-3D)	1.72814	1.68625	-	1.64199	1.58430	1.56301
	Li et al. (2008) (3D)	1.72227	1.67437	-	1.63053	1.57037	1.55788
5	Present	1.83900	1.82902	1.78974	1.79866	1.74266	1.73181
	Present (FEM)	1.8355	1.8259	1.7869	1.7958	1.7402	1.7294
	Bessaim et al. (2013) (Quasi-3D)	1.84465	1.84456	-	1.82032	1.75972	1.75143
	Li et al. (2008) (3D)	1.84198	1.82611	-	1.78956	1.72726	1.72670
10	Present	1.83744	1.84126	1.80581	1.81826	1.76525	1.75627
	Present (FEM)	1.8338	1.8380	1.8028	1.8153	1.7626	1.7537
	Bessaim et al. (2013) (Quasi-3D)	1.84113	1.85489	-	1.83973	1.78163	1.77878
	Li et al. (2008) (3D)	1.84020	1.83987	-	1.80813	1.74779	1.74811

Table 14 Nondimensional critical buckling loads  $\bar{N}_{cr}$  of Al/Al<sub>2</sub>O<sub>3</sub> sandwich square plates subjected to uniaxial compressive load ( $\gamma = 0$ ) with homogeneous hardcore (b/h=10).

p	Theory	1-0-1	2-1-2	2-1-1	1-1-1	2-2-1	1-2-1
0	Present	13.02212	13.02212	13.02212	13.02212	13.02212	13.02212
	Present (FEM)	12.9682	12.9682	12.9682	12.9682	12.9682	12.9682
	Zenkour (2005b) (TSDT)	13.00495	13.00495	13.00495	13.00495	13.00495	13.00495
	Zenkour (2005b) (SSDT)	13.00606	13.00606	13.00606	13.00606	13.00606	13.00606
	Meiche et al. (2011) (HSDT)	13.00552	13.00552	13.00552	13.00552	13.00552	13.00552
0.5	Present	7.36402	7.94190	8.22498	8.43730	8.81046	9.21716
	Present (FEM)	7.30884	7.88926	8.1724	8.38593	8.76012	9.16671
	Zenkour (2005b) (TSDT)	7.36437	7.94084	8.22470	8.43645	8.80997	9.21681
	Zenkour (2005b) (SSDT)	7.36568	7.94195	8.22538	8.43712	8.81037	9.21670
	Meiche et al. (2011) (HSDT)	7.36380	7.94046	8.22471	8.43647	8.81029	9.21757
1	Present	5.17491	5.84707	6.19951	6.46995	6.95434	7.50888
	Present (FEM)	5.15299	5.82231	6.17511	6.44260	6.92653	7.47726
	Zenkour (2005b) (TSDT)	5.16713	5.84006	6.19394	6.46474	6.94944	7.50656
	Zenkour (2005b) (SSDT)	5.16846	5.84119	6.19461	6.46539	6.94980	7.50629
	Meiche et al. (2011) (HSDT)	5.16629	5.83941	6.19371	6.46450	6.94952	7.50719
5	Present	2.66706	3.04926	3.40832	3.58429	4.11636	4.73597
	Present (FEM)	2.65566	3.03618	3.39720	3.56894	4.10226	4.71581
	Zenkour (2005b) (TSDT)	2.65821	3.04257	3.40351	3.57956	4.11209	4.73469
	Zenkour (2005b) (SSDT)	2.66006	3.04406	3.40449	3.58063	4.11288	4.73488
	Meiche et al. (2011) (HSDT)	2.65679	3.04141	3.40280	3.57874	4.11157	4.73463
10	Present	2.49751	2.75383	3.09685	3.20006	3.71221	4.28184
	Present (FEM)	2.48699	2.74203	3.08704	3.18635	3.70008	4.26359
	Zenkour (2005b) (TSDT)	2.48727	2.74632	3.09190	3.19471	3.70752	4.27991
	Zenkour (2005b) (SSDT)	2.48928	2.74844	3.13443	3.19456	3.14574	4.38175
	Meiche et al. (2011) (HSDT)	2.48574	2.74498	3.09111	3.19373	3.70686	4.27964

Table 15 Nondimensional critical buckling loads  $\bar{N}_{cr}$  of Al/Al<sub>2</sub>O<sub>3</sub> sandwich square plates subjected to biaxial compressive load (  $\gamma = 1$  ) with homogeneous hardcore (b/h=10).

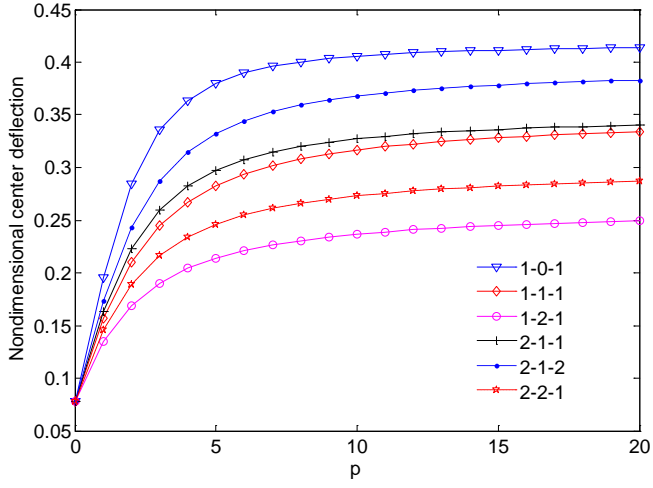
p	Theory	1-0-1	2-1-2	2-1-1	1-1-1	2-2-1	1-2-1
0	Present	6.51106	6.51106	6.51106	6.51106	6.51106	6.51106
	Present (FEM)	6.48413	6.48413	6.48413	6.48413	6.48413	6.48413
	Zenkour (2005b) (TSDT)	6.50248	6.50248	6.50248	6.50248	6.50248	6.50248
	Zenkour (2005b) (SSDT)	6.50303	6.50303	6.50303	6.50303	6.50303	6.50303
	Meiche et al. (2011) (HSDT)	6.50276	6.50276	6.50276	6.50276	6.50276	6.50276
0.5	Present	3.68201	3.97095	4.11249	4.21865	4.40523	4.60858
	Present (FEM)	3.65443	3.94464	408.623	4.19297	4.38007	4.58336
	Zenkour (2005b) (TSDT)	3.68219	3.97042	4.11235	4.21823	4.40499	4.60841
	Zenkour (2005b) (SSDT)	3.68284	3.97097	4.11269	4.21856	4.40519	4.60835
	Meiche et al. (2011) (HSDT)	3.68190	3.97023	4.11236	4.21823	4.40514	4.60878
1	Present	2.58746	2.92354	3.09976	3.23497	3.47717	3.75444
	Present (FEM)	2.57650	2.91116	3.08756	3.22130	3.46327	3.73864
	Zenkour (2005b) (TSDT)	2.58357	2.92003	3.09697	3.23237	3.47472	3.75328
	Zenkour (2005b) (SSDT)	2.58423	2.92060	3.09731	3.23270	3.47490	3.75314
	Meiche et al. (2011) (HSDT)	2.58315	2.91970	3.09686	3.23225	3.47476	3.75359
5	Present	1.33353	1.52463	1.70416	1.79214	2.05818	2.36798
	Present (FEM)	1.32783	1.51809	1.69860	1.78447	2.05113	2.35791
	Zenkour (2005b) (TSDT)	1.32910	1.52129	1.70176	1.78978	2.05605	2.36734
	Zenkour (2005b) (SSDT)	1.33003	1.52203	1.70224	1.79032	2.05644	2.36744
	Meiche et al. (2011) (HSDT)	1.32839	1.52071	1.70140	1.78937	2.05578	2.36731
10	Present	1.24875	1.37692	1.54843	1.60003	1.85611	2.14092
	Present (FEM)	1.24350	1.37102	1.54352	1.59318	1.85004	2.13180
	Zenkour (2005b) (TSDT)	1.24363	1.37316	1.54595	1.59736	1.85376	2.13995
	Zenkour (2005b) (SSDT)	1.24475	1.37422	1.56721	1.59728	1.57287	2.19087
	Meiche et al. (2011) (HSDT)	1.24287	1.37249	1.54556	1.59687	1.85343	2.13982

Table 16 Nondimensional fundamental frequency  $\bar{\omega}$  of Al/Al<sub>2</sub>O<sub>3</sub> sandwich square plates with homogeneous hardcore (b/h=10)

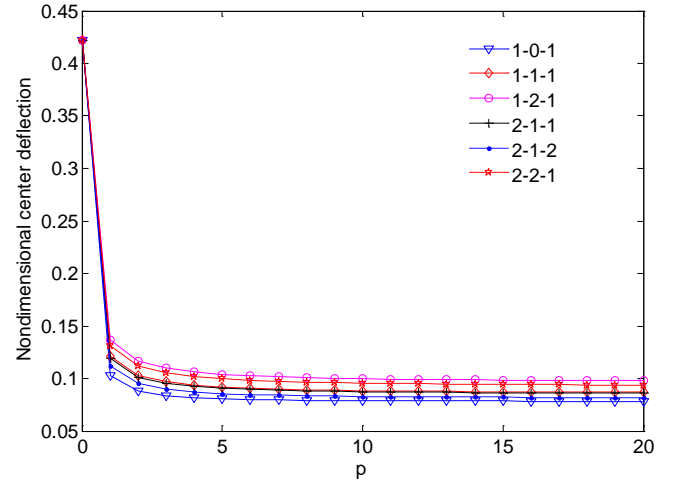
Boundary conditions	p	1-0-1	2-1-2	2-1-1	1-1-1	2-2-1	1-2-1
CCCC	0	3.0607	3.0607	3.0607	3.0607	3.0607	3.0607
	0.5	2.4499	2.5174	2.5533	2.5747	2.6200	2.6632
	1	2.1264	2.2225	2.2780	2.3095	2.3797	2.4477
	5	1.6287	1.6963	1.7824	1.8017	1.9145	2.0146
	10	1.5926	1.6310	1.7174	1.7200	1.8360	1.9310
CSCS	0	2.5514	2.5514	2.5514	2.5514	2.5514	2.5514
	0.5	2.0301	2.0865	2.1172	2.1351	2.1737	2.2107
	1	1.7573	1.8370	1.8837	1.9102	1.9693	2.0275
	5	1.3426	1.3956	1.4668	1.4832	1.5763	1.6618
	10	1.3154	1.3414	1.4129	1.4149	1.5104	1.5916
CFCF	0	1.9034	1.9034	1.9034	1.9034	1.9034	1.9034
	0.5	1.5161	1.5581	1.5812	1.5942	1.6231	1.6504
	1	1.3130	1.3725	1.4078	1.4269	1.4715	1.5142
	5	1.0038	1.0437	1.0984	1.1090	1.1800	1.2420
	10	0.9831	1.0032	1.0583	1.0581	1.1310	1.1897

Table 17 Nondimensional critical buckling loads  $\bar{N}_{cr}$  of Al/Al<sub>2</sub>O<sub>3</sub> sandwich square plates subjected to biaxial compressive load ( $\gamma = 1$ ) with homogeneous hardcore (b/h=10).

Boundary conditions	p	1-0-1	2-1-2	2-1-1	1-1-1	2-2-1	1-2-1
CCCC	0	15.2246	15.2246	15.2246	15.2246	15.2246	15.2246
	0.5	8.7735	9.4631	9.7877	10.0403	10.4708	10.9348
	1	6.2480	7.0559	7.4727	7.7877	8.3578	8.9901
	5	3.2493	3.7411	4.1883	4.3886	5.0455	5.7565
	10	3.0216	3.3833	3.8104	3.9287	4.5659	5.2201
CSCS	0	11.5415	11.5415	11.5415	11.5415	11.5415	11.5415
	0.5	6.5888	7.1089	7.3583	7.5484	7.8783	8.2338
	1	4.7621	5.2773	5.5941	5.8310	6.2640	6.7468
	5	2.4203	2.7781	3.1126	3.2619	3.7528	4.2921
	10	2.2575	2.5110	2.8306	2.9167	3.3918	3.8871
CFCF	0	8.6682	8.6682	8.6682	8.6682	8.6682	8.6682
	0.5	4.9125	5.3016	5.4924	5.6328	5.8839	6.1517
	1	3.4720	3.9225	4.1643	4.3376	4.6662	5.0277
	5	1.7933	2.0537	2.3092	2.4129	2.7845	3.1826
	10	1.6765	1.8554	2.0999	2.1557	2.5154	2.8795

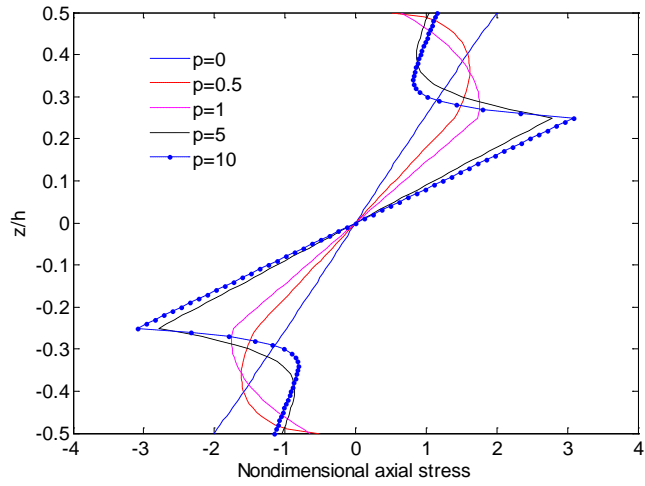


(a) Homogeneous hardcore

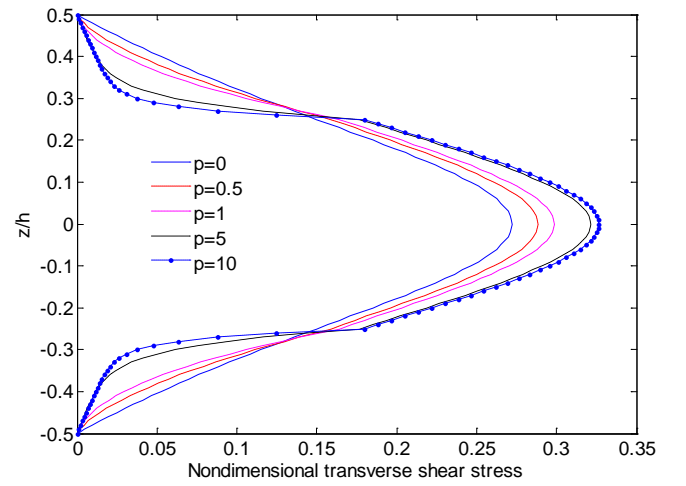


(b) Homogeneous softcore

Fig. 3 Effect of the power-law index  $p$  on the nondimensional center deflections ( $\bar{u}_3$ ) of Al/Al<sub>2</sub>O<sub>3</sub> sandwich square plates (b/h=10)

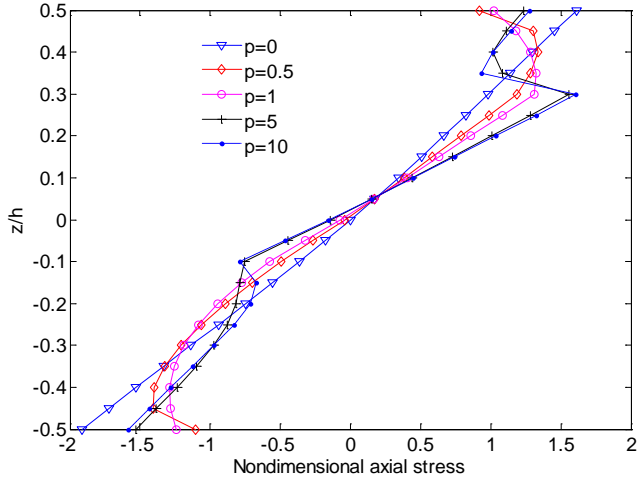


(a) Nondimensional axial stress  $\bar{\sigma}_{xx}$

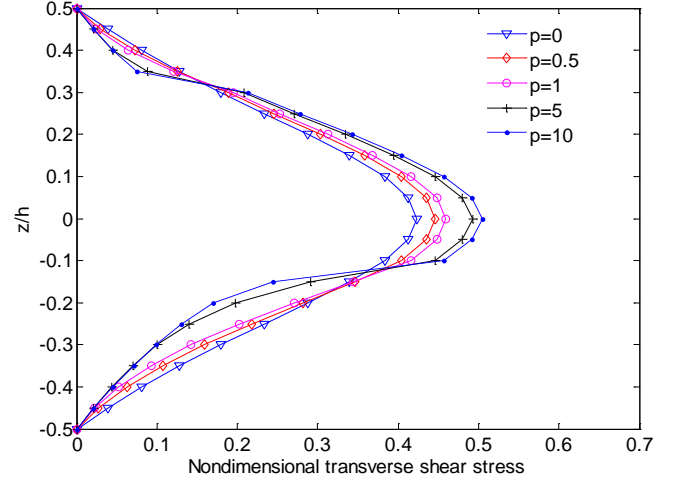


(b) Nondimensional shear stress  $\bar{\sigma}_{xz}$

Fig. 4 Variation of the nondimensional axial and shear stresses through the thickness of (1-2-1) Al/Al<sub>2</sub>O<sub>3</sub> sandwich square plates with homogeneous hardcore (b/h=10).

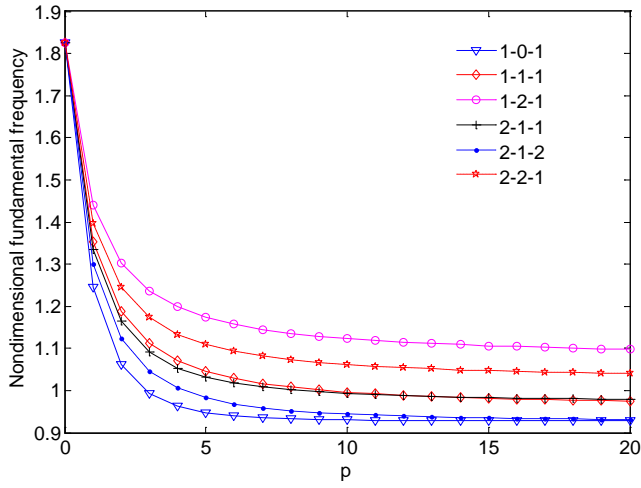


(a) Nondimensional axial stress  $\bar{\sigma}_{xx}$

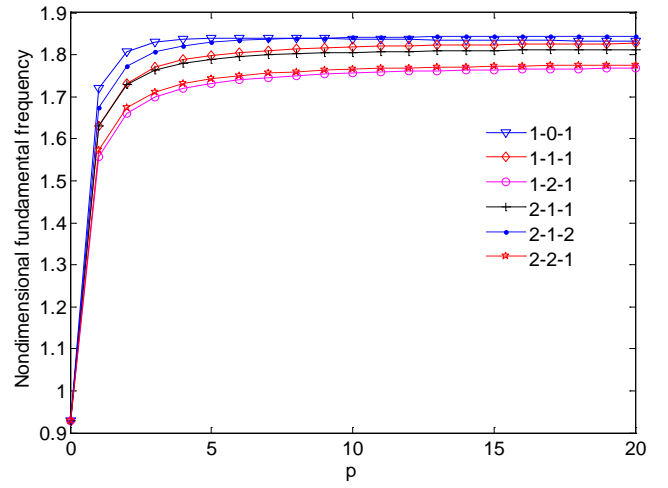


(b) Nondimensional shear stress  $\bar{\sigma}_{xz}$

Fig. 5 Variation of the nondimensional axial and shear stresses through the thickness of (2-2-1) Al/ZrO<sub>2</sub> CCCC sandwich square plates with homogeneous hardcore (b/h=10).

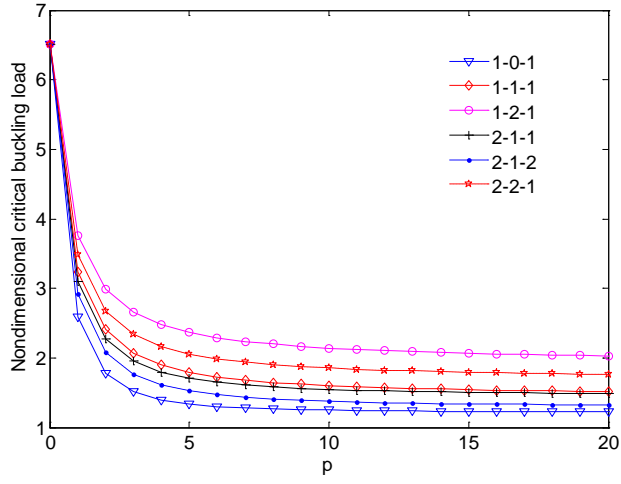


(a) Homogeneous hardcore

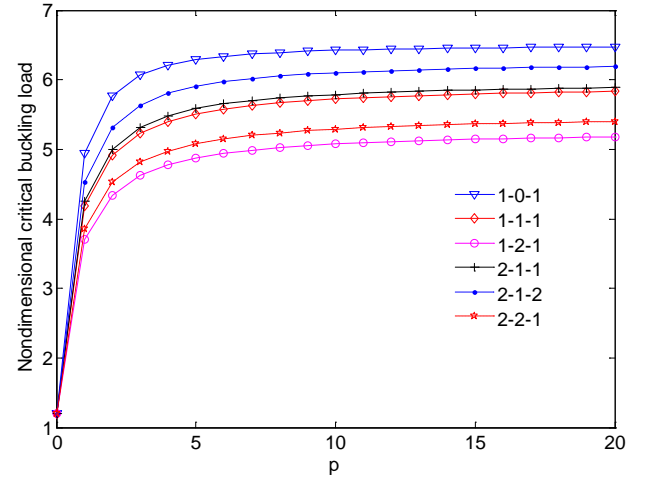


(b) Homogeneous softcore

Fig. 6 Effect of the power-law index  $p$  on the nondimensional fundamental frequency ( $\bar{\omega}$ ) of Al/Al<sub>2</sub>O<sub>3</sub> sandwich square plates (b/h=10).

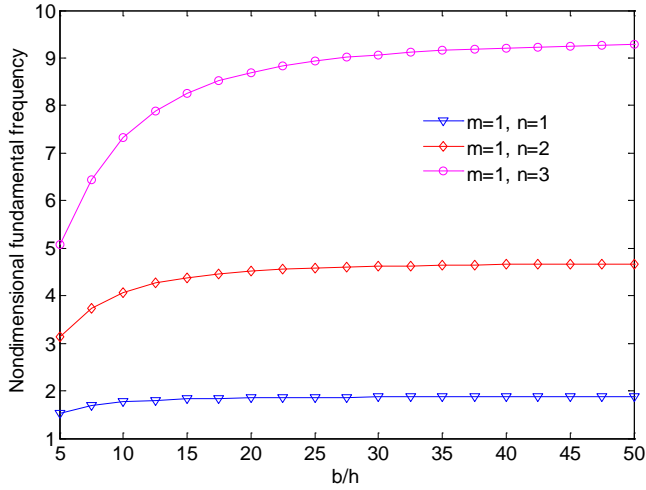


(a) Homogeneous hardcore

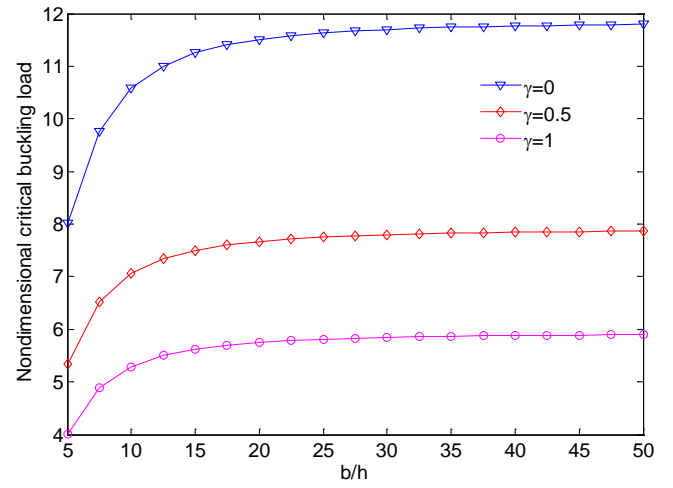


(b) Homogeneous softcore

Fig. 7 Effect of the power-law index  $p$  on the nondimensional critical buckling loads ( $\bar{N}_{cr}$ ) of Al/Al<sub>2</sub>O<sub>3</sub> sandwich square plates ( $b/h=10$ ) under biaxial compressions ( $\gamma = 1$ ).



(a) Nondimensional fundamental frequency



(b) Nondimensional critical buckling

Fig. 8 Effect of the side-to-thickness ratio  $b/h$  on the nondimensional fundamental frequency ( $\bar{\omega}$ ) and critical buckling load ( $\bar{N}_{cr}$ ) of (2-2-1) sandwich square plates with homogeneous softcore ( $p = 10$ ).